STREAM DEPLETION SPREADSHEET CONSTRUCTION

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Abstract: A stream-depletion solution obtained in 2003 and made available since then to practicing professionals is used to create an Excel spreadsheet example that calculates both stream depletion and drawdown in an observation well. Histories of aquifer types and of the stream-depletion problem are sketched to place the solution in context. Variable definitions for user-defined functions are given before showing how to implement these user-defined functions in spreadsheet calculations. Results are plotted, and a process is shown that fits the plotted curves to field data to estimate various aquifer parameters that appear in the solution. Finally, it is explained how the calculated results can be generalized to include stream depletion by multiple wells, to model more complicated pumping schedules, to compute stream depletion from specified stream reaches with finite lengths, to apply the results to cases in which streams have numerous reaches that gain or lose flow and to cases in which initial piezometric head distributions are not horizontal before pumping begins.

Introduction

For the past seven years the writer has made available a collection of user-defined programs for carrying out groundwater calculations with Excel spreadsheets. This collection of programs, entitled Function.xls, and a manual documenting its programs may be downloaded without charge from the writer's web site (http://www.civil.canterbury.ac.nz/staff/bhunt.shtml). During this time, however, enquiries have been received from some people requesting additional worked examples that show how to implement the programs in Function.xls. Since most of these enquiries have been concerned with stream depletion problems, in which pumping from a well beside a stream reduces flow in the stream, the following has been written to help people who want to use Function.xls but are not well acquainted with user-defined functions and Excel spreadsheet calculations in stream depletion problems. The writer wishes to emphasize that material contained herein is meant to supplement but not substitute for the study of an appropriate text on spreadsheet calculations. Readers who want to learn more about spreadsheet calculations and the construction and use of user-defined functions are referred to Liengme (2009).

Delayed-Yield Aquifer History

The spreadsheet programs used herein were constructed for what is often referred to as a delayed-yield aquifer. Because delayed-yield aquifers are not as widely used and understood as other aquifer types, a brief history of delayed-yield aquifers will be given before giving a history of the stream depletion problem.

Theis (1935) published the first solution for flow to a well in an aquifer that extends to infinity in all horizontal directions. This solution can be used to describe flow in a completely confined aquifer, when the top boundary is impermeable, or, as a first approximation, to describe flow in an unconfined aquifer, when the top boundary is a free surface open to the atmosphere. Since the Theis aquifer has no recharge source, drawdowns in the Theis solution continue to increase indefinitely as long as well abstraction continues.

Field experiments later showed that drawdowns in some aquifers appear to approach constant steady-flow values when pumped for sufficiently long periods of time. Hantush and Jacob (1955) addressed this problem by publishing the first solution for flow to a well in a leaky aquifer. A leaky aquifer is an aquifer underlain by a less permeable aquitard, which in turn is underlain by a highly permeable aquifer that maintains zero drawdown for all values of time.

Since zero drawdowns exist in the deeper aquifer for all time, this aquifer continues to recharge the upper leaky aquifer for all future values of time and allows steady-flow conditions to be reached in the upper pumped aquifer. This is sometimes referred to as the assumption of infinite storage.

No aquifer can supply limitless recharge to a pumped aquifer for an infinite period of time, and field experiments show that any leaky aquifer pumped for a sufficiently long period of time eventually has drawdowns that start to increase again at times greater than the period of quasi-steady flow that appears as true steady flow in the Hantush-Jacob solution. Boulton (1954, 1963) addressed this problem by obtaining an empirically derived solution that behaved in a way that was consistent with field observations. Ten years later Boulton (1973) and Cooley and Case (1973) showed that the apparently empirical solution of Boulton (1954, 1963) has a physical basis and actually describes flow to a well in a semi-confined aquifer. More recently Hunt (2003) gave an alternative proof that shows an identical result, namely that the Boulton delayed-yield solution describes flow to a well when the pumped aquifer is overlain by a less permeable aquitard containing a free surface. Downward recharge occurs initially from the overlying unconfined aquitard when pumping begins, and this causes drawdown values to appear to approach a state of steady flow. At later periods of time, however, the free surface in the aquitard draws down and vertical recharge into the lower pumped aquifer slows. At this point the period of quasi-steady flow ends, and drawdowns start to increase indefinitely in the pumped aquifer. Hunt and Scott (2005) compared the Boulton (1954, 1963) solution with numerical MOFLOW solutions to put limits on the transmissivity and specific yield of the aquitard in order that the Boulton solution is applicable.

There are several relevant notes that should be inserted at this point. First, there is only one known way in practice in which an infinite amount of aquifer recharge can be supplied to a pumped aquifer for an infinite period of time. This occurs when a body of surface water is connected through a system of aquifers and aquitards to a pumped aquifer. This is true, however, only if the body of surface water does not dry. Second, although the Boulton solution is not the most general solution possible (for example, it does not allow for the existence of bodies of surface water, for aquifer characteristics that change with spatial coordinates or relatively permeable layers above or below the pumped aquifer), the Boulton solution is more general than either the Theis (1935) or Hantush-Jacob (1955) solutions. This is because the delayed-yield solution can reproduce both the Theis and Hantush-Jacob solutions. In particular, the Theis solution is reproduced from the Boulton solution when the hydraulic conductivity of the overlying aquitard is set equal to zero, and the Hantush-Jacob solution is reproduced when the specific yield of the overlying aquitard is set equal to infinity.

Stream Depletion History

Theis (1941) published the first stream depletion solution, which gave the decrease in stream flow that occurred when a well abstracts water from a semi-infinite aquifer. The aquifer was the same type that was considered earlier by Theis (1935) in his solution for flow to a well. At that time the error and complimentary error functions apparently had not yet been defined, and the Theis solution was left in the form of a definite integral. Glover and Balmer (1954) later rewrote the Theis integral in terms of the complimentary error function, and from this point in history the Theis solution became more widely known as the "Glover-Balmer solution". Hantush (1965) obtained a solution that differed from the Theis (1941) solution by the presence of a less permeable aquitard lining the vertical stream boundary. Jenkins (1968) and Wallace et al. (1990)

used superposition and time translation with the Theis (1941) solution to obtain solutions for more general pumping schedules.

Most stream depletion problems occur in a context in which pumped aquifer boundaries more realistically can be assumed to extend to infinity on both sides of a stream that only partially penetrates water bearing strata. Hunt (1999) showed how to model this mathematically in a solution that placed an abstraction well on one side of a stream that partially penetrated an aquifer of the Theis (1935) type. The aquifer extended laterally to infinity in all directions, and the stream was modelled as an infinitely long straight line. Hunt (2003) later generalized this solution by obtaining a solution for the same well and stream geometry in a Boulton delayed-yield aquifer, which is the solution considered herein. This solution can be used to duplicate the Hunt (1999) solution by setting the hydraulic conductivity of the overlying aquitard equal to zero. It can also be used to calculate the solution for a Hantush-Jacob (1955) leaky-aquifer solution by setting the specific yield of the overlying aquitard equal to infinity. Since drawdown values in the Hantush-Jacob solution for flow to a well in a leaky aquifer decrease exponentially with distance from the pumped well, and since flow depletions from a stream decrease as drawdowns beneath the stream decrease, this modification causes steady-flow stream depletion values to decrease exponentially with distance between the stream and pumped well. However, this is both an inaccurate and unethical misuse of the solution since no underlying aquifer can be assumed to furnish an infinite amount of water for recharge into a pumped aquifer. More correctly, if the stream does not dry, and if the stream is the only body of surface water near the pumped aquifer, then the only possible steady-flow solution is one in which stream depletion and well abstraction are equal. The continuity (mass conservation) principle makes this result obvious, and it is a result contained in both of the Hunt (1999, 2003) solutions.

A solution has also been obtained by Hunt (2008) for a Boulton-type delayed-yield aquifer when the partially penetrating stream and aquifers on both sides of the stream all have finite widths. A user-defined program for this solution exists in Function.xls but is not used in the spreadsheets considered herein. The use of this program is explained in the manual "Groundwater Analysis Using Function", which describes all of the user-defined programs in Function.xls.

Although the solution used herein for a Boulton delayed-yield aquifer is capable of describing flow in either Theis or Hantush-Jacob type aquifers, it is worth pointing out that numerous experimental drawdown measurements have been published that show the reverses in curvature that typify flow in delayed-yield aquifers. Examples of this are given by Hunt et al (2001), Nyholm et al (2002), Kollet and Zlotnik (2003), Fox (2003, 2004), Lough (2005), and Lough and Hunt (2006).

Problem Description

Fig. 1 shows a definition sketch for flow to a well beside a stream. The following variables are defined in this sketch: Q = well abstraction rate, T = pumped aquifer transmissivity, S = pumped aquifer storativity, L = shortest distance between the well and stream, x = coordinate normal to the stream, y = coordinate parallel to the stream, z = vertical coordinate, K' = aquitard hydraulic conductivity, σ = aquitard specific yield, B' = aquitard saturated thickness, b = stream width and B" = aquitard thickness beneath the stream. Function.xls computes stream depletion with the following equation:

$$\frac{\Delta Q}{Q} = Q\_4\left(\frac{tT}{SL^2}, \frac{(K'/B')L^2}{T}, \frac{S}{\sigma}, \frac{\lambda L}{T}\right) \tag{1}$$

where ΔQ = stream depletion, t = time, λ = streambed resistance coefficient = K' b/B" and Q_4 is the call name for the user-defined program. Values for λ must usually be found by comparing measured and calculated drawdown values in an observation well. An example of this procedure is given in Lough and Hunt (2006). Drawdowns in the pumped aquifer are calculated in Function.xls with the following equation:

$$\frac{sT}{Q} = W\_4\left(\frac{x}{L}, \frac{y}{L}, \frac{tT}{SL^2}, \frac{\lambda L}{T}, \frac{(K'/B')L^2}{T}, \frac{S}{\sigma}\right) \tag{2}$$

where s = drawdown at the point (x, y) and W_4 is the call name for the user-defined program. All variable groupings in Eqs. (1) and (2) are dimensionless, so any consistent system of units may be used for calculations.

Spreadsheet Construction

Since user-defined functions are accessible only in Excel spreadsheets that have the required programs attached to a Visual Basic Editor, computations must be carried out either in Function.xls or in a copy of Function.xls that has been renamed. Therefore, open Function.xls and enter the data shown in rows 1 through 9 in Fig.2. Select cells A1:C2 and click on **Insert**, **Name** and **Create** to create the names a, b and n for the contents of cells A2, B2 and C2, respectively. Values for a, b and n allow n equally spaced points to be calculated on a $\log_{10}$ scale from 10^a to 10^b by using the formula

$$t_k = 10^{M_k} \quad (k = 1, 2, ...n) \tag{3}$$

where

$$M_k = a + (k-1)(b-a)/(n-1) \qquad (k = 1, 2, ...n) \tag{4}$$

Apply this formula by typing a 1 in cell A10, typing the equation

$$= A10 + 1 \tag{5}$$

in cell A11 and dragging the contents of cell A11 down until the value of n (in this case 50) is reached. (Dragging a formula downward is accomplished by left clicking on the bottom right corner of the cell containing the formula and dragging the cursor downward while keeping the left click depressed.) Then calculate the corresponding n values for t by entering in cell B10 the formula

$$= 10 \wedge (a + (A10-1) * (b-a)/(n-1)) \tag{6}$$

Duplicate this formula either by dragging it downward or by double clicking on the bottom right corner of cell B10. In this particular case, 50 points are calculated that are equally spaced on a $\log_{10}$ scale from t = 10^{-3} to t = 10^{3} days.

Name the cell values in cells B6:H6 by selecting cells B5:H6 and clicking on **Insert**, **Name** and **Create**. Then calculate the stream depletion, ΔQ, in cell C10 by typing in cell C10 the following formula, which is obtained from Eq.(1):

$$= Q * Q\_4(B10 * T/(S * L \wedge 2), KB * L \wedge 2/T, S/sigma, lambda * L/T) \tag{7}$$

where KB represents (K'/B') in Eq.(1). Since the variables Q, T, S, L, lambda, sigma and KB have been named, these variables may be typed directly into the equation in cell C10. An easier way, however, is to left click on the cell containing the value for a variable, and the name of that variable (rather than its value) will appear in the formula. Calculate all of the remaining values for ΔQ either by double clicking on the bottom right corner of cell C10 or by dragging the formula in cell C10 down to cell 59.

Plot the results just computed by selecting cells B10:C59 and clicking on **Insert**, **Chart** and **XY(Scatter)**. Then follow the four steps in the Chart Wizard by clicking on **Next** after completing each step. In step 4 the writer almost always prefers to embed the chart as an object in the sheet, as shown in Fig. 3. The chart may be modified after it has been made by right clicking on any part of the plot that needs to be changed and left clicking on any of the options that appear on the screen in a drop-down menu. For example, right clicking on the horizontal axis and choosing **Format Axis** allows the linear scale to be changed to a logarithmic scale.

Computations for drawdown in an observation well are carried out in a similar way and are shown in Fig. 4 for a well at the point (x, y) = (50, 100) m. This spreadsheet differs from the one shown in Fig. 3 only by the inclusion of the observation well coordinates in cells I6:J6 and by inserting in cell C10 the following formula:

$$= (Q/T)*W\_4\left(x/L, y/L, B10*T/(S*L\^{}2), lambda*L/T, KB*L\^{}2/T, S/\sigma\right) \qquad (8)$$

Field observations of drawdown can also be shown in the plot for Fig. 3. This is done by typing the field data in columns in a third sheet, selecting both columns of data and clicking on **Edit** and **Copy**. Then click on the plot embedded in sheet two (shown in Fig. 4) and click on **Edit** and **Paste Special**. This causes a drop-down menu to appear in which the user must ensure that "Add cells as **New series**", "Values (Y) in **Columns**" and "Categories (X Values) in **First Column**" are all ticked. Marker points used for the experimental points can be edited by right clicking on the plotted points and then left clicking on **Format Data Series**. Fig. 5 shows the end result for the example considered herein.

Values for aquifer parameters are obtained by adjusting values for T, S, lambda, sigma and KB for the calculated plot (cells C6, D6, F6, G6 and H6, respectively, in Fig. 4) until the calculated plot provides a good fit to the experimental points. (The calculated results and corresponding plot change almost instantaneously as soon as a new value is entered.) Fig. 6 shows the result of this fitting process, which is simplified considerably by realizing which parts of the plot are controlled by values for the different parameters. In particular, decreasing both K'/B' (= KB) and λ (lambda) moves segment BC in Fig. 5 upward, decreasing T increases the slope of segments AB and CD, decreasing S causes segment AB to translate leftward and decreasing σ (sigma) causes segment CD to translate leftward. A more complete discussion of this fitting process is given in Lough and Hunt (2006), where the purest can also see an example of fitting a calculated curve to actual field data rather than the artificial example shown in Figs. 5 and 6. Nevertheless, comparison of the fitted parameter values (T, S, σ, λ, K'/B') = (500, 0.0003, 0.017, 0.2, 0.0006) (units omitted for printing brevity) with the parameters (T, S, σ, λ, K'/B') = (530, 0.0003, 0.02, 0.1, 0.0006) that were actually used to artificially generate the "field data" suggests that λ is the most difficult parameter to determine accurately. This is because the biggest influence of λ is to determine the steady-flow asymptote of segment CD at large values of time, and the "experimental data" does not contain drawdown values for values of time that are sufficiently large to show this steady-flow asymptote.

Generalizations

All of the equations solved by Hunt (2003) to obtain the solution used herein are linear with coefficients that are independent of both spatial coordinates and time. These facts allow a number of generalizations to be made. For example, solutions for a number of single wells at different locations can be computed separately, and stream depletion values and drawdowns can be summed to obtain the solution for all of the wells abstracting water simultaneously. (It must be remembered, however, that drawdown values for each of the single-well solutions must first be referenced to one common coordinate system.) Since well abstraction in this solution is zero for all negative values of time and equal to a constant, Q, for all positive values of time, translating a second solution in time by an amount Δt and subtracting the result from a first solution with an identical abstraction rate starting at $t = 0$ gives the solution for a pulse in which the abstraction rate is Q for $0 < t < \Delta t$ and zero for all other positive and negative values of t. This same technique can be used to obtain a solution for any pumping schedule in which well abstraction is a different constant over specified finite ranges of time. Values of stream depletion per unit length of stream can be computed at any point along the stream length by multiplying the drawdown beneath the stream at that point by λ. Furthermore, integrating these values over any finite length of stream reach will give the stream depletion that occurs over that stream reach. Finally, although the solution was obtained for the case in which drawdowns and stream inflows and outflows (leakages) are all zero at $t = 0$, the result also applies to more general cases in which the piezometric surface is not horizontal and where different stream reaches may be gaining or losing water to the ground before pumping begins. In this case, drawdown values calculated from the solution are interpreted as changes in piezometric levels created by the abstraction well, and these changes must be subtracted from pre-existing piezometric levels to obtain the final piezometric head distribution. Likewise, stream depletion values calculated from the solution are interpreted as changes in stream flow from the pre-existing distribution of stream flow values before well abstraction begins. Often, however, changes computed from the solution are of more practical interest than the result of subtracting changes from pre-existing piezometric levels and stream depletion values.

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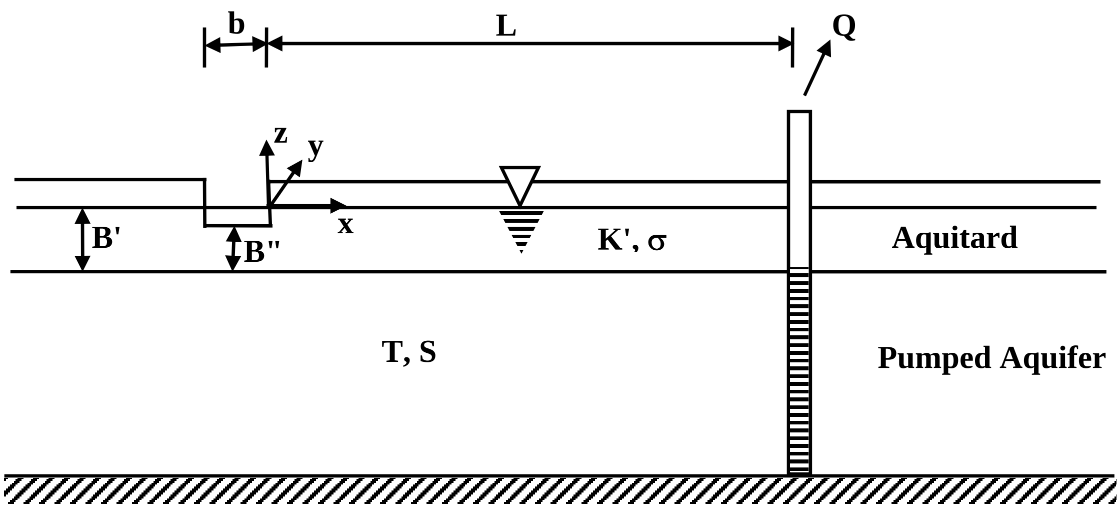

Fig.1. Flow to a well beside a stream

	A	B	C	D	E	F	G	H	I
1	**a**	**b**	**n**						
2	-3	3	50						
3									
4	**Units:**	**m^3/day**	**m^2/day**	**Dimless**	**m**	**m/day**	**Dimless**	**days**	
5	**Symbol:**	**Q**	**T**	**S**	**L**	**lambda**	**sigma**	**KB**	
6	**Value:**	3000	1000	0.0001	100	1	0.1	0.001	
7									
8		**Calculated Points for Plot**							
9	**k**	**t (days)**	**ΔQ (L/s)**						
10	1	0.001	57.54227						
11	2	0.001326	76.74802						
12	3	0.001758	99.69554						
13	4	0.00233	126.6653						
14	5	0.003089	157.9226						
15	6	0.004095	193.7029						
16	7	0.005429	234.1872						
17	8	0.007197	279.4666						
18	9	0.009541	329.4942						
19	10	0.012649	384.0231						
20	11	0.016768	442.5333						
21	12	0.02223	504.151						
22	13	0.029471	567.5742						
23	14	0.039069	631.0267						
24	15	0.051795	692.2799						

Fig. 2. Spreadsheet calculations for stream depletion

	A	B	C	D	E	F	G	H	I	J	K
1	**a**	**b**	**n**								
2	-3	3	50								
3											
4	**Units:**	**m^3/day**	**m^2/day**	**Dimless**	**m**	**m/day**	**Dimless**	**days**			
5	**Symbol:**	**Q**	**T**	**S**	**L**	**lambda**	**sigma**	**KB**			
6	**Value:**	3000	1000	0.0001	100	1	0.1	0.001			
7											
8		**Calculated Points for Plot**									
9	**k**	**t (days)**	**ΔQ (L/s)**								
10	1	0.001	57.54227								
11	2	0.001326	76.74802								
12	3	0.001758	99.69554								
13	[illegible]	[illegible]	[illegible]								
14											
15											
16											
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34											
35											
36											
37	[illegible]	[illegible]	[illegible]								
38	29	2.682696	913.6548								
39	30	3.55648	916.6604								
40	31	4.714866	920.6195								

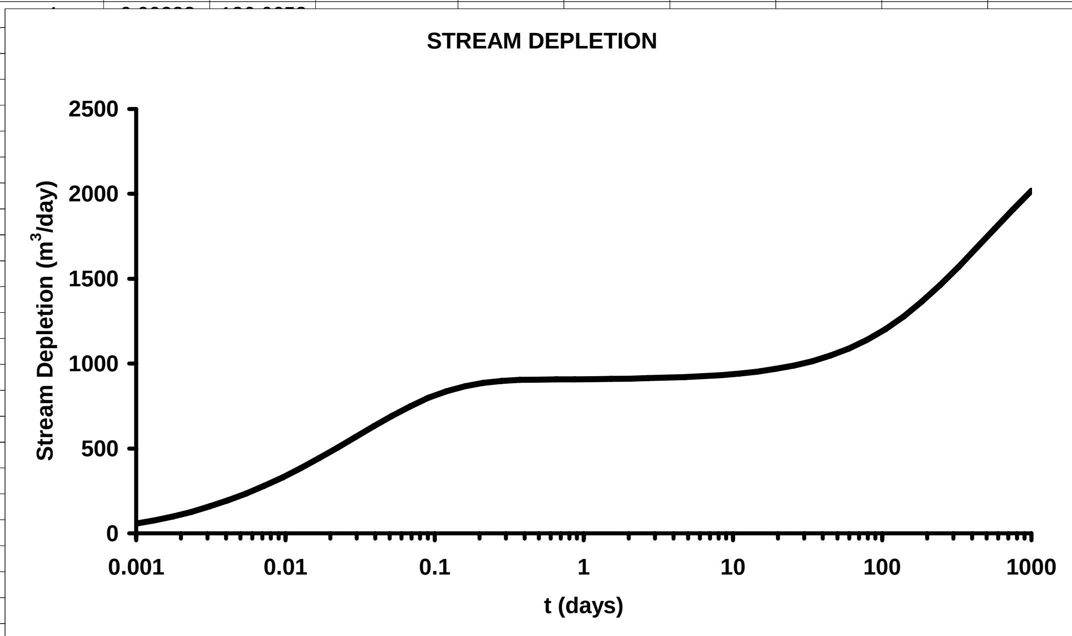
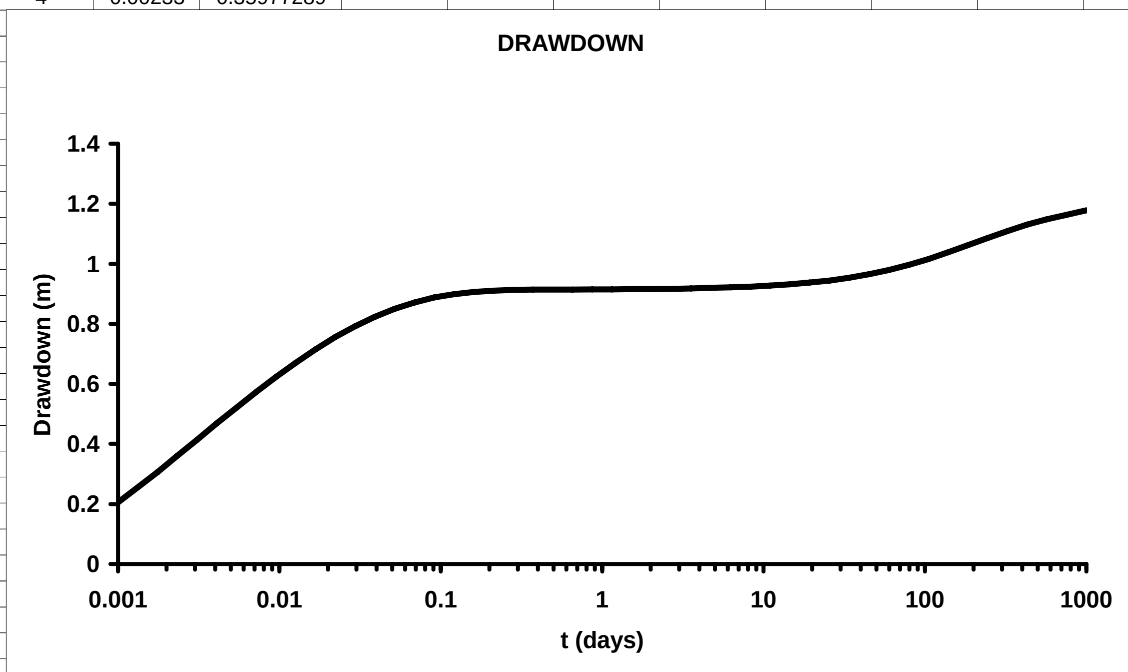


Fig. 3. A stream-depletion plot embedded in its worksheet

	A	B	C	D	E	F	G	H	I	J	K
1	**a**	**b**	**n**								
2	-3	3	50								
3											
4	**Units:**	**m^3/day**	**m^2/day**	**Dimless**	**m**	**m/day**	**Dimless**	**days**	**m**	**m**	
5	**Symbol:**	**Q**	**T**	**S**	**L**	**lambda**	**sigma**	**KB**	**x**	**y**	
6	**Value:**	3000	1000	0.0001	100	1	0.1	0.001	50	100	
7											
8		**Calculated Points for Plot**									
9	**k**	**t (days)**	**Dwdown (m)**								
10	1	0.001	0.205676506								
11	2	0.001326	0.254761892								
12	3	0.001758	0.306444139								
13	4	0.00233	0.35977289								



	A	B	C	D	E	F	G	H	I	J	K
39	[illegible]	[illegible]	[illegible]								
40	31	4.714866	0.919563968								
41	32	6.250552	0.921520782								
42	33	8.286428	0.924085301								
43	34	10.98541	0.927435274								

Fig. 4. Drawdowns plotted for an observation well

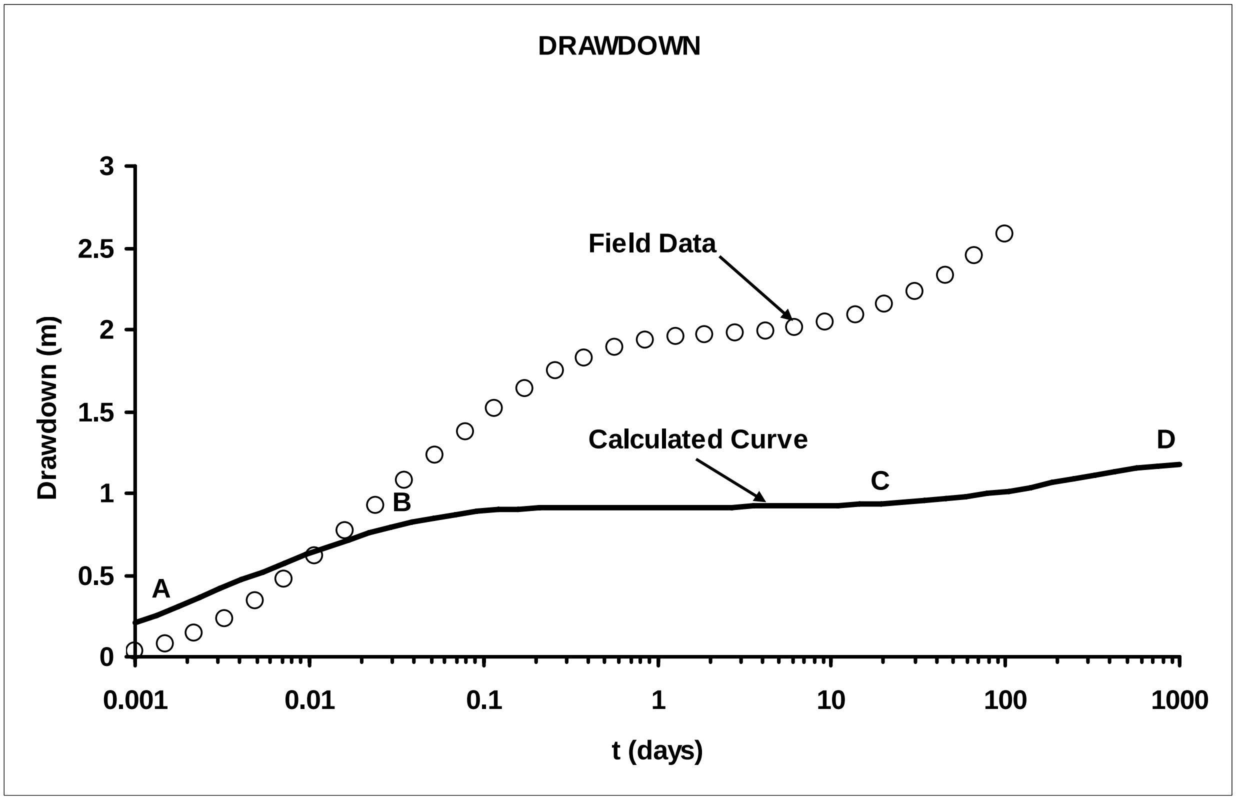


Fig. 5. Drawdown field data superimposed on the plot in Fig. 4

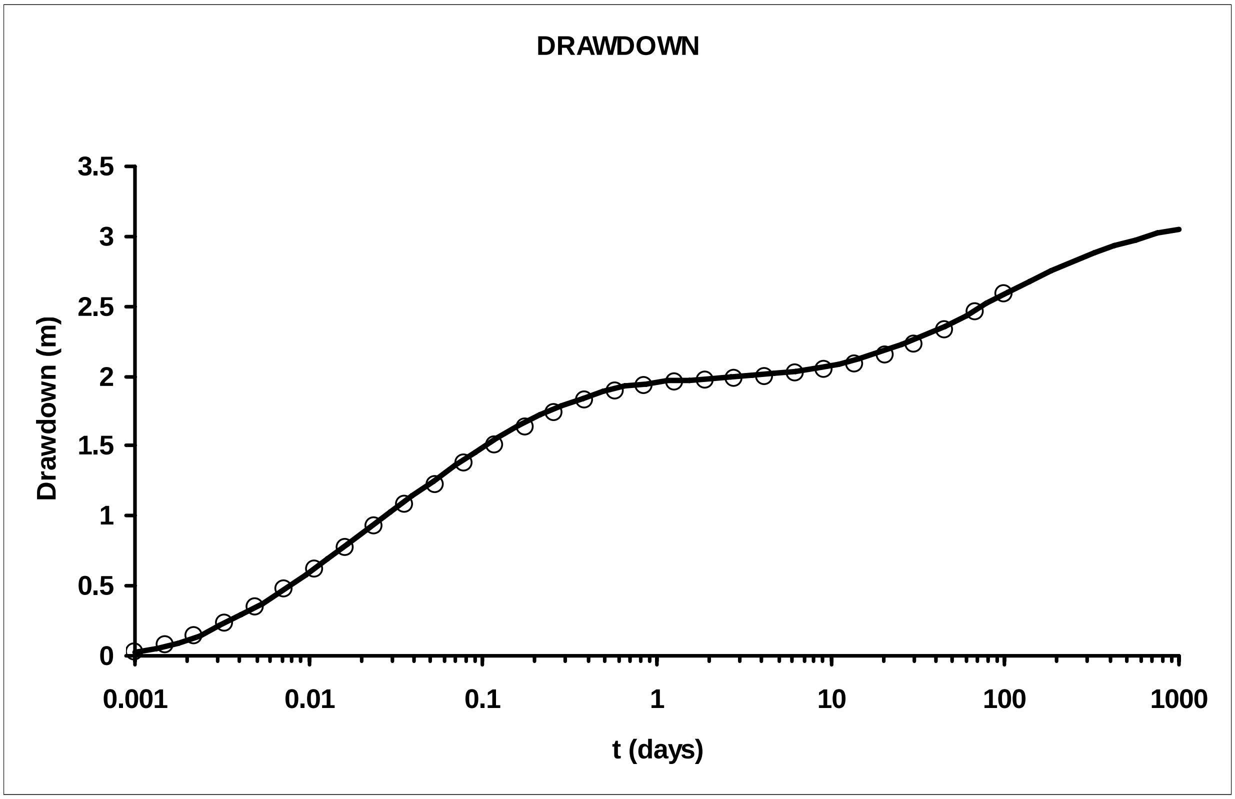


Fig. 6. Adjusted fit of field data in Fig. 5 to calculated curve.