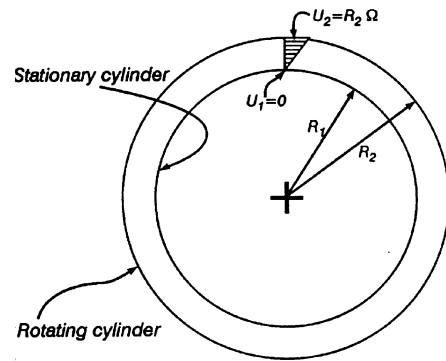


# CHAPTER 1

## INTRODUCTION PROBLEMS

- 1.1 A viscometer has fluid contained in the annular region between two concentric cylinders. The outer cylinder rotates with an angular velocity  $\Omega$ , and the torque  $T$  that is required to hold the inner cylinder stationary is measured.



- (a) Assume a linear velocity distribution across the annular region to obtain an algebraic relationship between the torque, angular velocity, viscosity, cylinder height and the two radii. You will need to use Eq. (1.1).
- (b) Water at  $10^\circ\text{C}$  is tested in the viscometer. Then air at  $20^\circ\text{C}$  is tested. Calculate the ratio of torque for water to torque for air if  $\Omega$  is the same in both tests. Viscosities for water and air are given in Appendix I.

Note: You may find it interesting to know that the inner cylinder must be held stationary while the outer cylinder is rotated. If the outer cylinder is fixed while the inner cylinder is rotated, the flow may become unstable. This means that a system of vortices may occur within the annular region, and the assumption of a linear velocity distribution will no longer apply

- 1.2 In example 3.2 it is shown that the height of capillary climb for water at  $5^\circ\text{C}$  in a 5 mm diameter tube is 6.15 mm. Calculate this height if the temperature is increased to  $30^\circ\text{C}$ . Surface tensions for water are given in Appendix I.

Note: Impurities in water can cause marked changes in surface tension. In fact, one standard way to reduce capillary climb heights in the laboratory is to add a drop or two of ordinary detergent to each manometer tube. Therefore, you should not interpret calculated heights of capillary climb too literally.

- 1.3 Mercury is an example of a fluid that does not 'wet' a glass surface. As a result, a mercury meniscus is concave downward instead of concave upward as in example 3.2. What does this mean in terms of pressures beneath the meniscus and the height of capillary climb for a mercury manometer like the one shown in example 3.2?

- 1.4 Appendix I shows that water cavitates (boils) at  $100^\circ\text{C}$  at an absolute pressure of  $101.3 \times 10^3 \text{ N/m}^2$ . Since water boils at  $100^\circ\text{C}$  at sea level, this tells us that absolute atmospheric pressure at sea level is  $101.3 \times 10^3 \text{ N/m}^2$ , which is a gage pressure of  $0 \text{ N/m}^2$ . Therefore, as stated in the footnote on page 1.4, vaporization **gage** pressures can be calculated from values given in Appendix I simply by subtracting from each absolute vapour pressure this value of  $101.3 \text{ kPa}$ . Use this method to calculate the gage pressure at which water cavitates for a temperature of  $10^\circ\text{C}$ . Then convert this pressure, in  $\text{N/m}^2$ , to an equivalent height of water, in m, by dividing the pressure by  $\rho g$ .
- 1.5 Calculate for the function  $\phi = (x^2 - y^2)t^3$  values for  $\partial\phi/\partial x$ ,  $\partial\phi/\partial y$  and  $\partial\phi/\partial t$ .
- 1.6 Differentiate first with respect to  $y$  and then with respect to  $x$  to obtain  $\partial^2 F/\partial x\partial y$  for  $F = x^2y^3$ . Next, differentiate first with respect to  $x$  and then with respect to  $y$  to obtain  $\partial^2 F/\partial y\partial x$ . Compare these two results to show that Eq. (1.27) holds for this particular function. In fact, Eq. (1.27) will always hold for any function.
- 1.7 A function,  $\phi$ , is defined by the following set of partial differential equations:

$$\frac{\partial\phi}{\partial x} = x + xy \quad \text{and} \quad \frac{\partial\phi}{\partial y} = -y + \frac{1}{2}x^2$$

A solution for a set of equations like this does not always exist. In fact, a solution will exist only if  $\partial^2\phi/\partial x\partial y = \partial^2\phi/\partial y\partial x$ . Differentiate the first equation with respect to  $y$  and the second equation with respect to  $x$ . Then note that the two results are equal, so that a solution for  $\phi$  exists in this case. Finally, integrate these two equations to calculate  $\phi$ . Make the solution for  $\phi$  unique by requiring that  $\phi = 1$  at  $x = y = 0$ .

- 1.8 You will see in chapter 2 that Newton's second law, when viscous forces are neglected becomes

$$-\nabla p + \rho \mathbf{g} = \rho \mathbf{a}$$

where  $g$  points in the direction of gravity and  $|g| = g = 9.81 \text{ m/s}^2$  at sea level. In words, this vector equation states that the sum of pressure and gravitational forces per unit volume equals the product of the mass per unit volume with the acceleration. Let the  $y$  axis point upward in a flow so that  $\mathbf{g} = -g\hat{j}$ , and let  $p = x^2y + x^3$ . Calculate  $\rho \mathbf{a}$ , which is a vector that points in the direction of the acceleration vector,  $\mathbf{a}$ .

1.9 In general, the vector velocity field within a flow is a function of  $x$ ,  $y$ ,  $z$  and  $t$ .

$$\mathbf{V} = \mathbf{V}(x, y, z, t)$$

If we follow the motion of a particular fluid particle whose  $(x, y, z)$  coordinates change with  $t$ , then  $x = x(t)$  and  $z = z(t)$ . Thus, the acceleration of this fluid particle is calculated from the following ‘chain rule’:

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{\partial\mathbf{V}}{\partial x} \frac{dx(t)}{dt} + \frac{\partial\mathbf{V}}{\partial y} \frac{dy(t)}{dt} + \frac{\partial\mathbf{V}}{\partial z} \frac{dz(t)}{dt} + \frac{\partial\mathbf{V}}{\partial t}$$

Since  $dx/dt$ ,  $dy/dt$  and  $dz/dt$  are seen from Eq. (1.17) to be the fluid particle velocity components  $u$ ,  $v$ , and  $w$ , this result becomes

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = u \frac{\partial\mathbf{V}}{\partial x} + v \frac{\partial\mathbf{V}}{\partial y} + w \frac{\partial\mathbf{V}}{\partial z} + \frac{\partial\mathbf{V}}{\partial t}$$

The sum of the four terms on the right occurs so often in the study of fluid motion that it is called the ‘material derivative’ or the ‘derivative following the motion of a fluid particle’ and is written

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt} = \left( \mathbf{V} \cdot \nabla + \frac{\partial}{\partial t} \right) \mathbf{V}$$

where

$$\begin{aligned} \mathbf{V} \cdot \nabla &= (u\hat{i} + v\hat{j} + w\hat{k}) \cdot \left( \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \\ &= u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \end{aligned}$$

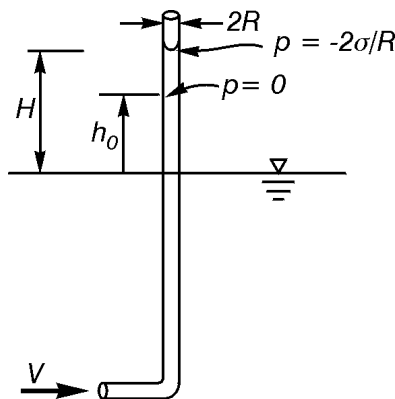
Use this result to calculate an expression for the acceleration vector if  $\mathbf{V} = x\hat{i} - y\hat{j}$ . Then use Eq. (1.20) to calculate the streamlines for this flow in the region  $0 < x < \infty$  and  $0 < y < \infty$ . Note that this velocity field models flow in a corner if flow is allowed to ‘slip’ along the physical boundaries, which are the positive  $x$  axis and the positive  $y$  axis. This kind of flow is known as potential or irrotational flow and is studied in chapter 6. Streamlines for this flow are sketched in the solution of this problem.



## CHAPTER 3

## FLUID STATICS PROBLEMS

- 3.1 Fluid pressure has units in the SI system of  $\text{N/m}^2$  ( $= \text{kg/m-s}^2$ ), which are sometimes called pascals. However, it is also common in engineering to describe a pressure as the depth below a free surface in a particular fluid at which that pressure occurs. Use this information to carry out the following calculations:
- Convert a pressure of  $30,000 \text{ N/m}^2$  to the equivalent height of water and mercury. The mass density of water is  $1000 \text{ kg/m}^3$ , and the specific gravity of mercury is 13.6 at  $10^\circ\text{C}$ . The gravitational constant,  $g$ , is  $9.81 \text{ m/s}^2$  at sea level.
  - A discussion of absolute pressures, gage pressures and vaporization pressures is given under the heading *Flow Properties* in Chapter 1. Reread this material. Then calculate the equivalent height of water, in gage pressure, for a pressure of absolute zero at sea level. For a U.S. Standard Atmosphere, this pressure is  $-1.013 \times 10^5 \text{ N/m}^2$  gage.
  - Use the information in Appendix I to calculate, in gage pressure, the equivalent height of water at which water vaporizes at  $10^\circ\text{C}$  at sea level.
- 3.2



A stagnation tube, which consists of a small diameter hollow tube with a right angle bend, is inserted into a river to measure the flow velocity directly in front of the stagnation opening. The principles introduced in Chapter 4 can be used to show that this velocity is given by

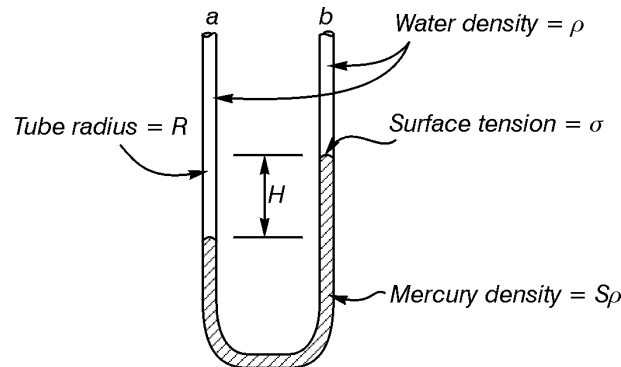
$$V = \sqrt{2gh_0}$$

in which  $h_0$  = elevation of the point of zero pressure above the river free surface. However, the tube has a relatively small radius,  $R$ . Thus, surface tension,  $\sigma$ , causes a negative pressure of

$$p = -2\sigma/R$$

at the free surface within the tube. If the free surface within the tube is a distance  $H$  above the river free surface, calculate the velocity in terms of  $H$ ,  $\sigma$  and  $R$ . Then use this equation and surface tension data given in the appendix to calculate the range for  $R$  that would make errors in  $V$  less than two per cent if the surface tension effect is neglected. Assume a water temperature of  $10^\circ\text{C}$  and a value for  $H$  of 0.1 m.

3.3

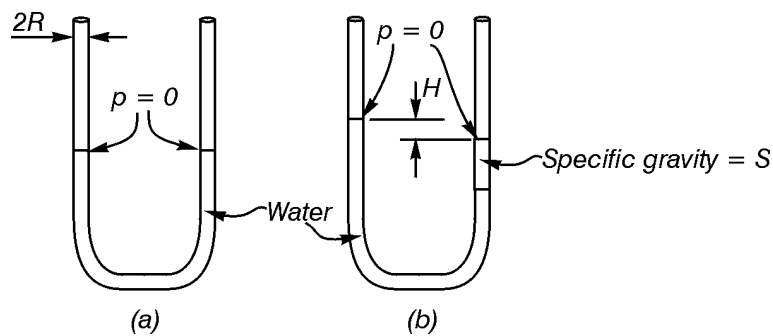


A water-mercury manometer is used to measure the pressure difference across a flow measuring device. A discontinuity in pressure of

$$|\Delta p| = 2\sigma/R$$

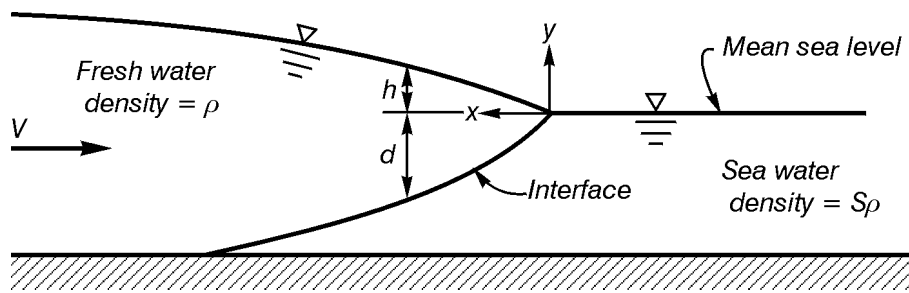
occurs across the water-mercury interface as a result of surface tension, with the higher pressure occurring on the concave side of the interface. Determine an expression for  $(p_a - p_b)$  if  $H$  is known.

3.4



Water is at rest in a U-tube, as shown in (a). Then a fluid of specific gravity  $S$  is poured into the right side of the tube, and the system is allowed to come to rest. Calculate the difference in free surface elevation,  $H$ , if the volume of introduced fluid is  $\ell\pi R^2$ . The U-tube is open to the atmosphere and surface tension effects are negligible.

3.5



Salt water is heavier than fresh water. Consequently, when fresh water in a groundwater aquifer approaches a sea coast, the fresh water flows up over a wedge of relatively dense sea water. An approximation known as the Ghyben-Herzberg approximation assumes that:

- (1) The interface between fresh and sea water is approximated as a surface of discontinuity in density (the sharp interface approximation).
- (2) The sea water in the aquifer is everywhere at rest, so that sea water pressures are distributed hydrostatically everywhere.
- (3) Fresh water pressures are distributed hydrostatically along vertical lines. (The Dupuit approximation.)

Use these approximations to show that the depth,  $d$ , of the interface below mean sea level at any point is given by

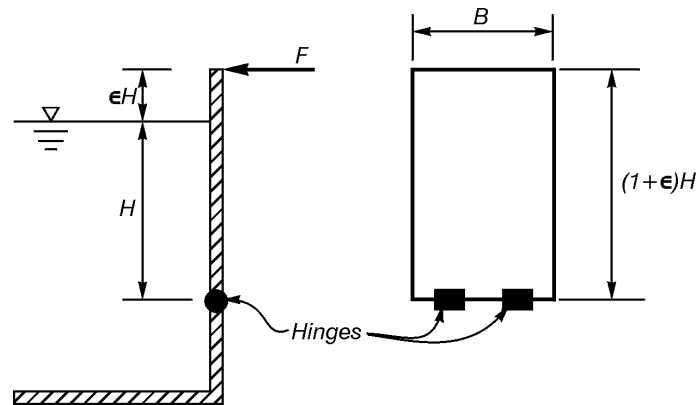
$$d = h / (S - 1)$$

in which  $S$  = specific gravity of sea water and  $h$  = elevation of the fresh water free surface above mean sea level at the point where  $d$  is being calculated. It is usually assumed that  $S \approx 1.025$ , which gives the easily remembered result

$$d \approx 40h$$

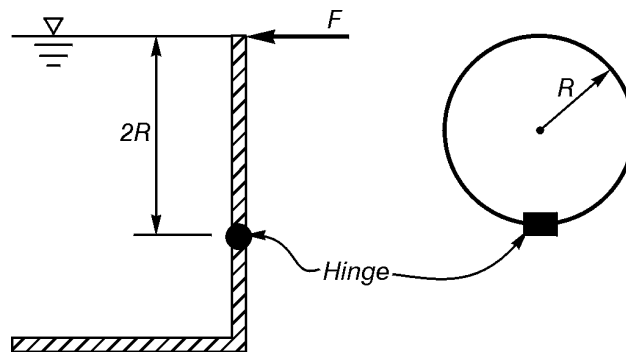
From this we see that, like an iceberg, most of the fresh water occurs below sea level.

3.6



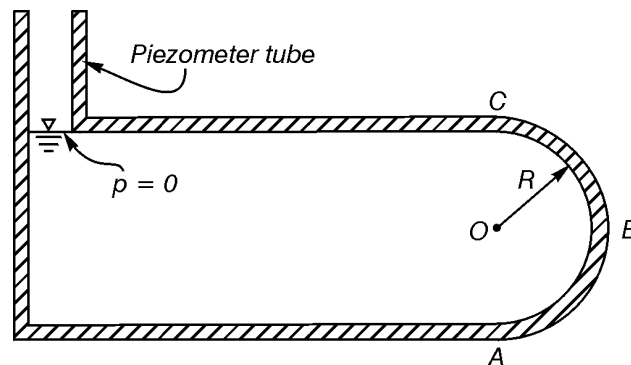
Calculate the minimum force,  $F$ , that is required to hold the rectangular gate shut. Then calculate the hinge reaction force.

3.7



Calculate the minimum force,  $F$ , that is required to hold the circular gate shut.

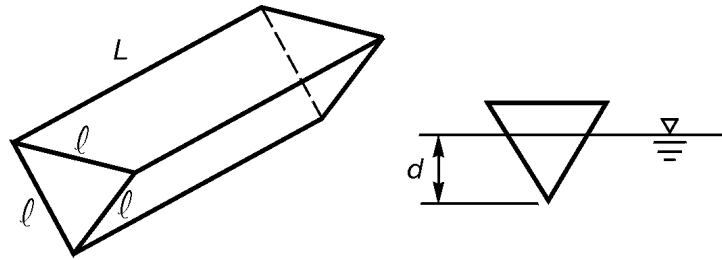
3.8



The rectangular tank of fluid has three vertical side walls. The fourth lateral boundary,  $ABC$ , is half of a right circular cylinder. Calculate magnitudes, directions and lines of action for the vertical and horizontal pressure forces per unit width on the wetted surface  $ABC$ .

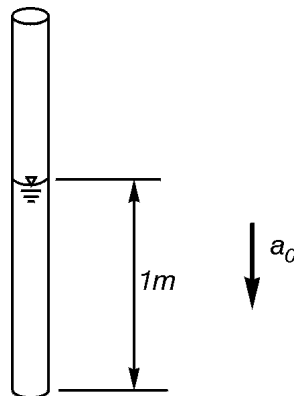
3.9 Repeat the calculation in problem 3.8 if the water level in the piezometer tube has an elevation  $kR$  above point  $C$ .

- 3.10 Show that the summation of moments about point  $O$  of the vertical and horizontal components of the pressure forces on  $ABC$ , in problem 3.8 vanishes. Then, after considering the direction of the pressure force acting on a small segment of the arc  $ABC$ , explain why the moment about point  $O$  of these pressure forces must vanish.
- 3.11 A homogeneous sphere with a specific gravity  $S$  (with  $S < 1$ ) is tethered with a rope to the bottom of a reservoir of fresh water. Calculate the tensile force in the rope if the sphere is completely submerged. The sphere has a diameter of  $D$ .
- 3.12



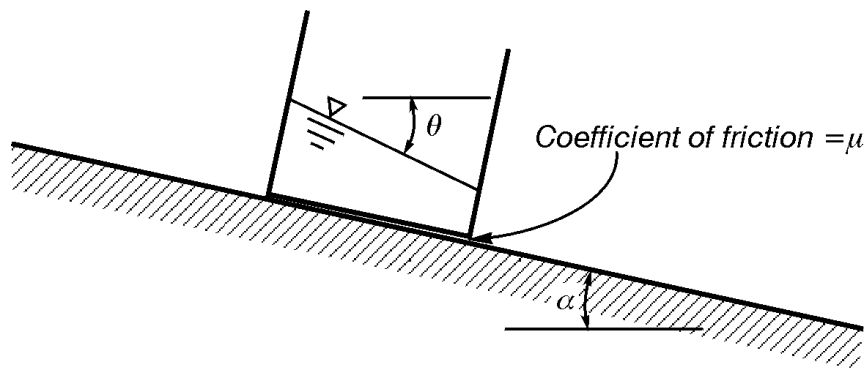
A homogeneous solid object (specific gravity =  $S$  with  $S < 1$ ) floats in a reservoir of fresh water. The object has a length  $L$  and a cross section in the shape of an equilateral triangle with sides of length  $l$ . As suggested by the sketch,  $L > l$ . Calculate the submergence depth,  $d$ .

- 3.13 Determine the required range for values of  $S$  in problem 3.12 if the object is to be stable in the position shown.
- 3.14



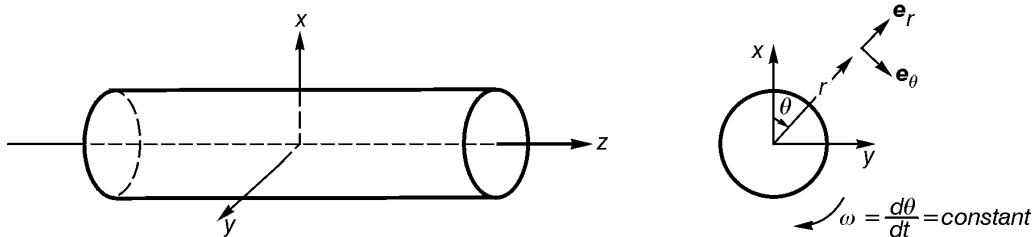
Water stands at a depth of 1 m in a tube that is open to the atmosphere at its top. Calculate the downward acceleration,  $a_0$ , that is just sufficient to cause cavitation at the bottom of the tube when the water temperature is  $10^\circ\text{C}$ . (Cavitation occurs when the water vaporizes, as discussed in Chapter 1 on page 1.4.)

3.15



A tank of water, which is open to the atmosphere, slides down a plane surface that is inclined at an angle  $\alpha$  to the horizontal. The fluid and container have a total mass of  $M$ , and the coefficient of sliding friction between the tank bottom and plane is  $\mu$ . Calculate the constant acceleration vector of the tank and fluid. Then calculate the angle,  $\theta$ , that the free surface makes with the horizontal if the fluid moves as a rigid body. Finally, use the requirement that the downslope weight component must exceed the friction force for motion to occur to obtain  $\sin \alpha > \mu \cos \alpha$ . Show that this requires  $\theta > 0$ , and then give a physical explanation for this result.

3.16

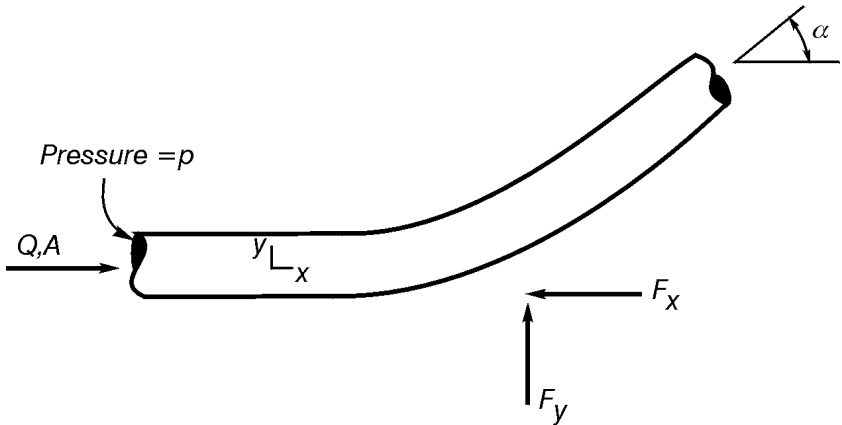


Fluid in a cylindrical tank spins as a rigid body about a horizontal axis with a constant angular velocity  $d\theta/dt = \omega$ . Obtain expressions for  $\partial p/\partial r$ ,  $\partial p/\partial \theta$  and  $\partial p/\partial z$ . Then show that these equations have a solution by showing that  $\partial^2 p/\partial \theta \partial r = \partial^2 p/\partial r \partial \theta$ ,  $\partial^2 p/\partial \theta \partial z = \partial^2 p/\partial z \partial \theta$  and  $\partial^2 p/\partial r \partial z = \partial^2 p/\partial z \partial r$ . Finally, calculate  $p$  and show in a drawing the position and geometry of curves of constant  $p$  relative to the tank boundary. Then comment on the possibility of using this method to manufacture concrete pipe.

# CHAPTER 4

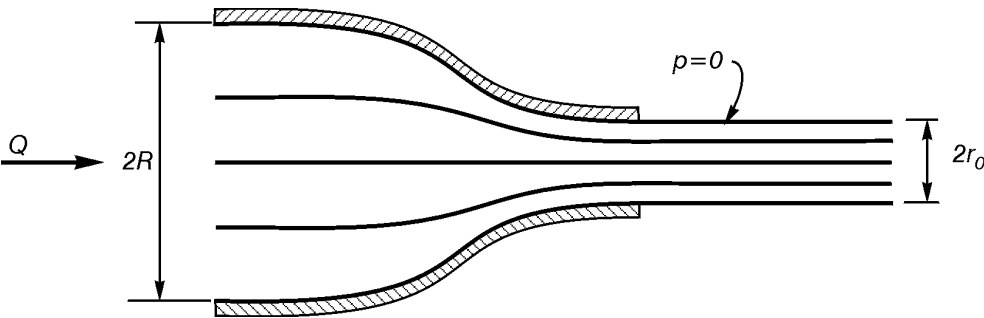
## CONTROL VOLUME METHODS PROBLEMS

4.1



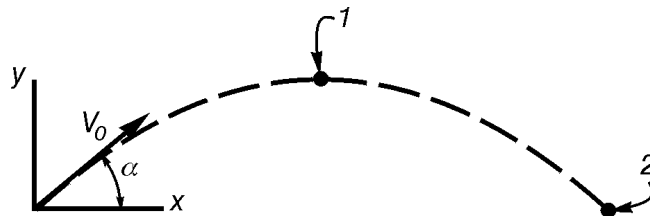
Flow in a constant diameter pipe has a known uniform flow pressure,  $p$ , a flow rate,  $Q$ , and an area,  $A$ . Calculate the force components  $F_x$  and  $F_y$  that the pipe walls must exert on this flow in order to turn the flow through an angle,  $\alpha$ , from its original direction. What would happen if the pipe is unable to provide these forces? For a flexible fire hose these force components must be supplied by a fireman.

4.2



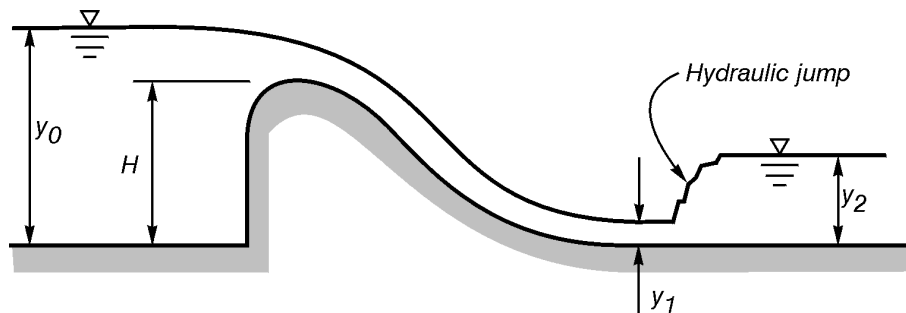
Flow passes through a streamlined fire hose nozzle and exits to the atmosphere as a free jet. The flow rate,  $Q$ , and the two radii,  $R$  and  $r_0$ , are known. What are the velocities at the entrance and exit of the nozzle?

- 4.3 Calculate the pressure at the nozzle entrance in problem 4.2. As explained in Example 4.3, the pressure is atmospheric in the free jet where streamlines are straight and parallel. Assume that the flow velocities are large enough to allow gravity to be neglected.
- 4.4 Calculate the longitudinal pressure force that the nozzle walls exert on the flow in problem 4.2
- 4.5



A free jet has an initial velocity  $V_0$  and is inclined at an angle  $\alpha$  to the horizontal. Neglect air resistance, as in Example 4.5, to calculate the jet trajectory in the form  $y = f(x)$ .

- 4.6 Calculate the  $x$  and  $y$  coordinates of points 1 and 2 in problem 4.5. Notice that the  $x$  coordinate of point 2 is twice the  $x$  coordinate of point 1. It can be shown that the jet trajectory calculated in problem 4.5 is a parabola that is symmetric about point 1.
- 4.7 What values of  $\alpha$  in problem 4.6 will give the maximum  $y$  coordinate of point 1 and the maximum  $x$  coordinate of point 2? Assume that all variables except  $\alpha$  are held constant.
- 4.8

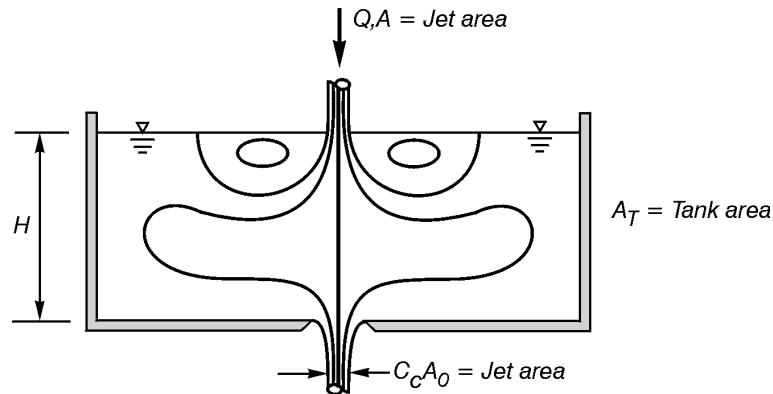


Flow over a spillway has a given discharge coefficient,  $C_D$ , that we will define by

$$q = C_D \sqrt{2g(y_0 - H)^3}$$

in which  $q$  = flow rate per unit width. Use the Bernoulli and continuity equations to obtain an expression for  $y_1/y_0$  in terms of  $C_D$  and  $H/y_0$ . (One unwanted root of the cubic equation for  $y_1/y_0$  can be removed in the same way that this is accomplished in Example 4.7, and the resulting quadratic equation has only one real positive root.)

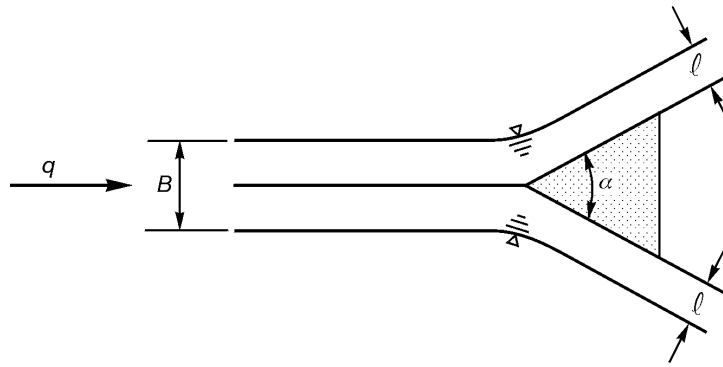
- 4.9 Suppose in problem 4.8 that  $y_0 = 15$  m,  $H = 10$  m and  $C_D = 0.500$ . Calculate the depth,  $y_1$ , the velocity,  $V_1$ , and the tail water depth,  $y_2$ , that must exist in the stilling basin to hold the hydraulic jump in place.
- 4.10 Calculate the horizontal force that the spillway exerts upon the flow in problem 4.8. Write the answer in terms of  $y_0$ ,  $y_1$  and  $q$ . Then put in the numbers used in problem 4.9.
- 4.11



A vertical free jet with a known flow rate,  $Q$ , and area,  $A$ , enters a tank of water. A circular orifice in the tank floor has an area,  $A_0$ , and allows a free jet with a known contraction coefficient,  $C_c$ , to exit in the downward direction. The orifice diameter is small compared with the water depth,  $H$ . The Bernoulli equation cannot be applied without a head loss term along the centre streamline because of large amounts of turbulence and energy dissipation that occur near the inflow. Near the tank walls, however, turbulence intensities and energy dissipation are relatively small. Apply the Bernoulli equation along a streamline that starts on the free surface at the tank wall and passes through the free surface in the contracted jet to calculate the water depth,  $H$ .

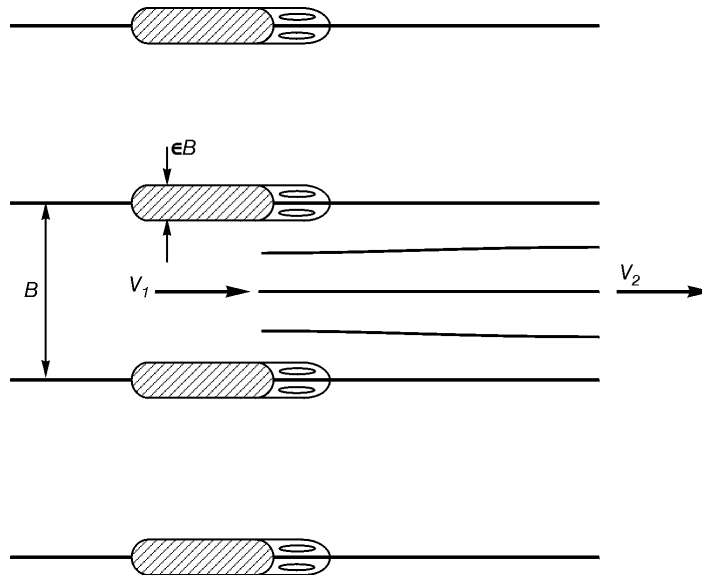
- 4.12 The tank support structure in problem 4.11 must be strong enough to support the sum of the empty tank weight and the force exerted by the fluid on the tank surface. Use the expression for  $H$  from problem 4.11 and the momentum equation to calculate the force exerted on the fluid by the tank surface.

4.13



A two-dimensional jet with a thickness,  $B$ , and flow rate per unit width,  $q$ , strikes a wedge. Assume that the jet speed is large enough to allow the neglect of gravity and that symmetry exists about a horizontal plane through the leading edge of the wedge. Show that the deflected jet thickness,  $l$ , is given by  $l = B/2$ . Then calculate the force per unit width exerted by the wedge upon the flow.

4.14



A plan view is shown for flow between bridge piers with a spacing  $B$  and a pier width  $\epsilon B$ . Since flow between any two adjacent piers is similar to the flow considered in Example 4.2, and since flow depths are constant in directions normal to the streamlines, similar methods of analysis can be used in both problems. Apply the continuity and momentum equations between cross sections 1 and 2 to obtain

$$\left(\frac{y_2}{y_1}\right)^3 - \left(\frac{y_2}{y_1}\right) = 2(1 - \epsilon) \left[ \left(\frac{y_2}{y_1}\right) - 1 + \epsilon \right] F_1^2$$

in which  $F_1 = \frac{V_1}{\sqrt{gy_1}} = \text{Froude number}$

$y_1, y_2 = \text{flow depths}$

- 4.15 The cubic equation in problem 4.14 can be solved for  $\epsilon \ll 1$  by noting that  $(y_2/y_1) = 1$  is the solution of physical interest when  $\epsilon = 0$ . Thus, a perturbation solution for  $(y_2/y_1)$  can be calculated in the following form:

$$\left( \frac{y_2}{y_1} \right) = 1 + \epsilon \phi_1 + \epsilon^2 \phi_2 + \dots$$

Substitute this into the cubic equation and equate coefficients of  $\epsilon$  to obtain

$$\phi_1 = \frac{F_1^2}{1 - F_1^2}$$

This shows that  $y_2 > y_1$  if  $F_2 < 1$  (subcritical flow) and that  $y_2 < y_1$  if  $F_1 > 1$  (supercritical flow). However, standing waves cause this one-dimensional solution to become inaccurate for supercritical flow.

- 4.16 Flow separation and turbulence downstream from cross section 1 in problem 4.14 prevented us from using the Bernoulli equation in this region. Upstream from cross section 1, however, energy losses are negligible. Therefore, apply the Bernoulli and continuity equations between point 0 upstream from the piers and point 1 in cross section 1 to obtain

$$\left( \frac{y_1}{y_0} \right)^2 - \left( \frac{y_1}{y_0} \right)^3 = \frac{1}{2} \left[ \frac{1}{(1 - \epsilon)^2} - \left( \frac{y_1}{y_0} \right)^2 \right] F_0^2$$

in which

$$F_0 = \frac{V_0}{\sqrt{g y_0}}$$

$y_0, y_1$  = flow depths

- 4.17 Since  $\epsilon \ll 1$  and  $y_1/y_0 = 1$  is the solution of physical interest when  $\epsilon = 0$ , a perturbation solution of the cubic equation in problem 4.16 can be calculated in the form

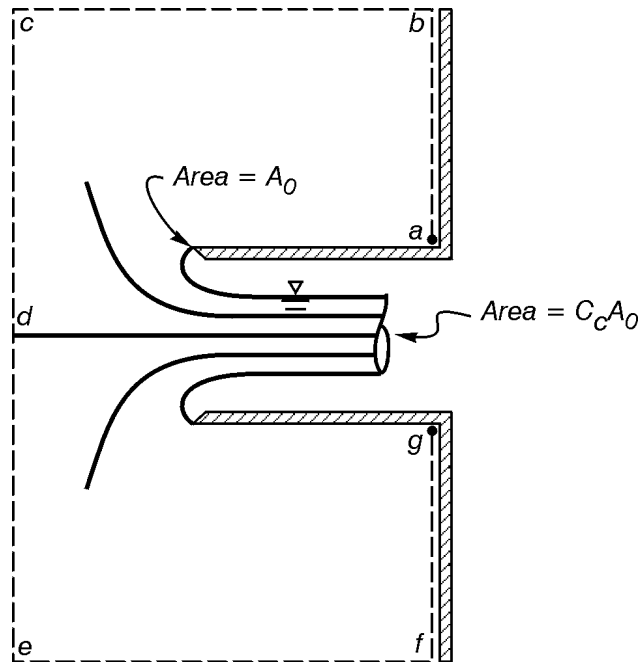
$$\left( \frac{y_1}{y_0} \right) = 1 + \epsilon \phi_1 + \epsilon^2 \phi_2 + \dots$$

Show that

$$\phi_1 = - \frac{F_0^2}{1 - F_0^2}$$

Thus,  $y_1 < y_0$  when  $F_0 < 1$  and  $y_1 > y_0$  when  $F_0 > 1$ . Standing waves in supercritical flow, however, make this one-dimensional solution inaccurate when  $F_0 > 1$ .

4.18

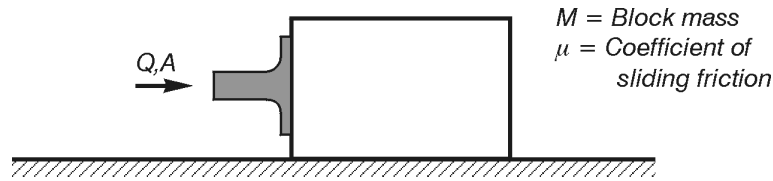


Control volume methods can be used to calculate the contraction coefficient for a reentrant tube (Borda's mouthpiece!) in a large reservoir. This requires use of both the momentum and Bernoulli equations. If gravity is neglected, then pressures along the dashed line *abcdefg* are a constant,  $p_0$ , since  $V \rightarrow 0$  at all points along this line. Therefore, the net horizontal force on the control volume is  $p_0 A_0$ . Use this result in the momentum equation. Then use the Bernoulli equation to calculate  $p_0$ , and, ultimately, show that

$$C_c = \frac{1}{2}$$

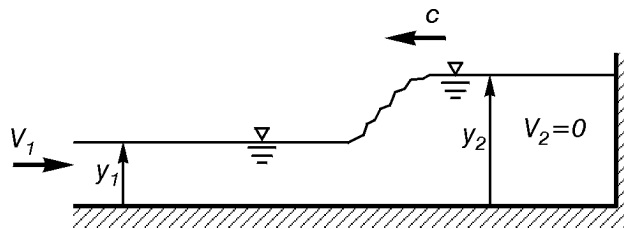
This result holds for both a slot and a circular orifice.

4.19



A high-speed liquid jet strikes the side of a block. Calculate the constant terminal block speed.

4.20



A landslide completely blocks a river that initially had a velocity and depth of  $V_1$  and  $y_1$ , respectively. Choose a control volume that moves upstream with the constant surge speed,  $c$ , and use the momentum and continuity equations to show that

$$F_1 = \left( y_2/y_1 - 1 \right) \sqrt{\frac{1}{2} \left( 1 + \frac{1}{y_2/y_1} \right)}$$

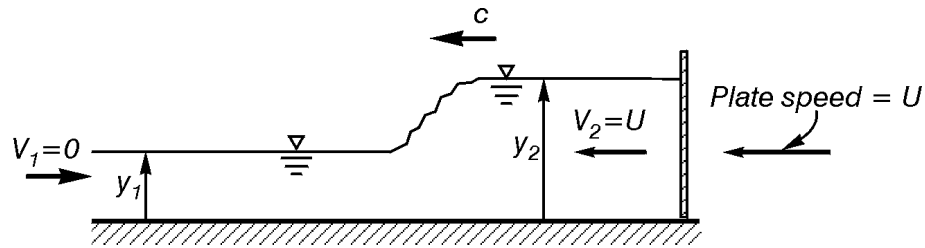
$$\frac{c}{\sqrt{g y_1}} = \sqrt{\frac{1}{2} \left( 1 + \frac{1}{y_2/y_1} \right)}$$

in which

$$F_1 = \frac{V_1}{\sqrt{g y_1}} = \text{Froude number}$$

Calculate  $y_2$  and  $c$  for a flow that had  $y_1 = 1$  m and  $V_1 = 10$  m/s.

4.21



A motionless reservoir of water has a depth  $y_1$ . A vertical plate then creates a surge by moving through the reservoir with a constant speed,  $U$ . Show that the surge celerity and depth,  $y_2$ , can be calculated from

$$\frac{c}{\sqrt{gy_1}} = \sqrt{\frac{1}{2} \frac{y_2}{y_1} \left( \frac{y_2}{y_1} + 1 \right)}$$

$$F_1 = \frac{(y_2/y_1 - 1)}{y_2/y_1} \sqrt{\frac{1}{2} \frac{y_2}{y_1} \left( \frac{y_2}{y_1} + 1 \right)}$$

in which

$$F_1 = \frac{U}{\sqrt{gy_1}} = \text{Froude number}$$

Calculate  $y_2$  and  $c$  if  $y_1 = 1$  m and  $U = 1$  m/s.

# CHAPTER 5

## DIFFERENTIAL EQUATION METHODS PROBLEMS

5.1 Fully developed flow in Fig. 7.1 is a solution of Eq. (7.2):

$$g \frac{dh(x)}{dx} = \nu \frac{D^2 u(y)}{dy^2}$$

Choose  $u \sim U_{\max}$  = centreline velocity and  $y \sim B/2$  to obtain an estimate for  $\Delta h$  over a distance  $x$ . Then compare this with the following exact result calculated in Chapter 7:

$$\Delta h = 8 \frac{\nu x U_{\max}}{g B^2}$$

5.2 Consider the region of developing flow between two flat plates that is shown in Fig. 7.1. Equation (5.21) gives an estimate for the boundary-layer thickness

$$\frac{\delta}{L} \sim \frac{1}{\sqrt{U_{\max} L / \nu}}$$

in which  $L$  = distance downstream from the reservoir entrance and  $U_{\max}$  = maximum velocity, which occurs at a distance  $\delta$  from the boundary. The exact Blasius solution for development of a boundary layer along a flat plate gives

$$\frac{\delta}{L} = \frac{5}{\sqrt{U_{\max} L / \nu}}$$

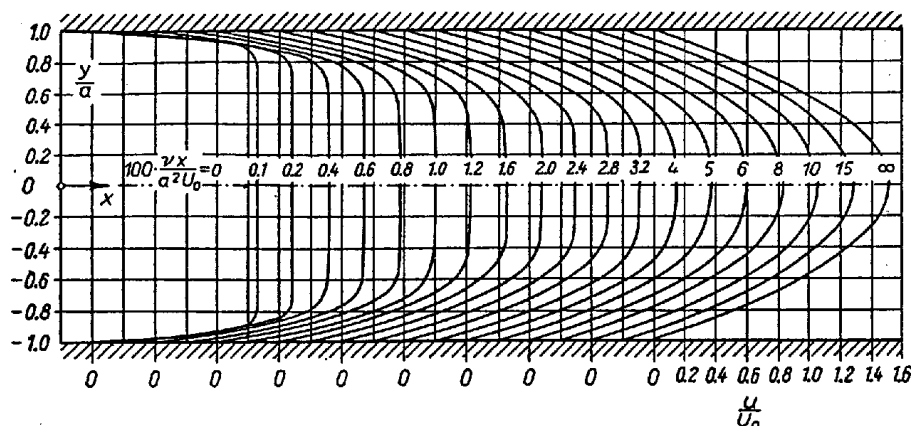
The Blasius solution neglects changes in both pressure and  $U$  that occur in Fig. 7.1. Nevertheless, set  $\delta = B/2$  and use Eq. (7.9) to obtain the following estimate for the distance downstream where flow becomes fully developed:\*

$$\frac{L}{B} = 0.015 Re \left( Re = \frac{UB}{\nu} \right)$$

Then compare this result with an approximate solution obtained by Schlichting (1968) that is plotted below.

---

\* Flow becomes fully developed at the point where  $\delta = B/2$ . Thereafter, velocity profiles cease to change with  $x$ .



Reproduced from:

Schlichting, H. (1968) *Boundary-layer Theory*, 6th edition, McGraw-Hill Book Co., New York, pp. 176-178.

Each velocity distribution in this figure has a different origin that can be determined by noticing that the velocity is zero on the top and bottom boundaries. The velocity scale is given at the bottom right hand corner of the plot.

5.3 Creeping flow occurs when accelerations are negligible compared with the viscous terms.

(a) Consider viscous flow past a sphere for which  $\ell_x \sim \ell_y \sim \ell_z \sim D =$  sphere diameter. Write down the equations in three dimensions that correspond to (5.1) - (5.2). [Look at Eqs. (2.4), (2.20), (2.15) and (2.28) if these equations do not appear obvious to you.] From the continuity equation, obtain estimates for  $V$  and  $W$  in terms of  $U =$  velocity of approach at infinity.

(b) From the momentum equation, show that the creeping flow approximation requires

$$Re \ll 1 \text{ in which } Re = \frac{UD}{\nu}$$

$$p \sim \mu U/D \text{ in which } p = \text{pressure change}$$

(c) The surface area and volume of a sphere are  $\pi D^2$  and  $\pi D^3/6$ , respectively. Thus, show that the pressure, viscous and total forces on the sphere have the estimates

$$F_{\text{pressure}} \sim \pi D \mu U$$

$$F_{\text{viscous}} \sim \pi D \mu U/6$$

$$F = F_{\text{pressure}} + F_{\text{viscous}} \sim \frac{7}{6} \pi D \mu U$$

(d) In 1851 Stokes obtained the solution for creeping flow past a sphere. The drag force is given by Eq. (7.14), and the drag coefficient is compared with experimental data in Fig. 7.3. The experimental solution, of course, includes acceleration terms. How do your estimates  $Re \ll 1$  and  $F \sim 7\pi D \mu U/6$  compare with these results?

- (e) Methods used in this chapter do not guarantee the existence of solutions. For example, no solution exists for creeping flow past a two-dimensional circular cylinder. This is because a logarithmic singularity in the two-dimensional solution does not allow uniform flow to be approached at infinity. It can be argued that this is not the fault of the order of magnitude estimates, since the creeping flow approximation only holds when  $r \sim D$  and cannot be expected to apply when  $r \rightarrow \infty$ . Nevertheless, there is no other physically obvious boundary condition that can be applied when  $r \sim D$ , and we are left without a solution in this case. This is known as Stoke's paradox. Approximate solutions have been obtained for slow viscous motion past both a sphere and circular cylinder by approximating the acceleration term. This procedure is known as the Oseen approximation.



## CHAPTER 6

### IRROTATIONAL FLOW PROBLEMS

6.1 Examine Figs. 4.4 - 4.5 and read the discussion concerning these figures in Chapter 4. Then discuss why the irrotational flow approximation is much more accurate for highly accelerated flow than for decelerated flow.

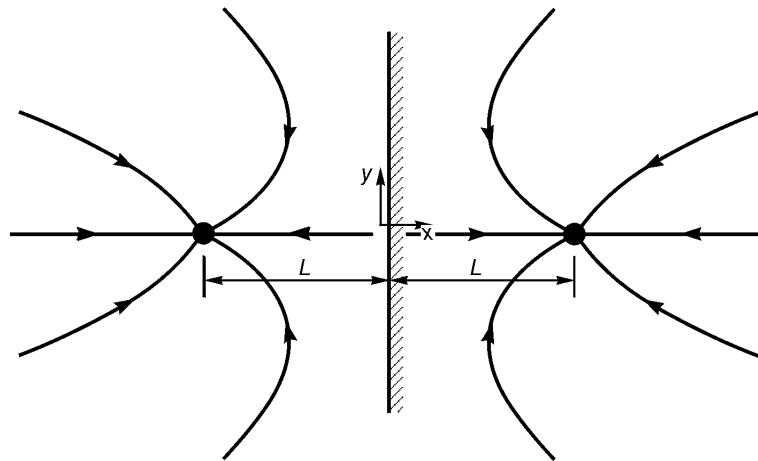
6.2 Two-dimensional irrotational flow in a corner has the velocity potential

$$\phi = \frac{1}{2} C (x^2 - y^2) \text{ m}^2/\text{s}$$

in which  $C$  is a constant.

- (a) Show by direct differentiation that this expression for  $\phi$  satisfies Eq. (6.15).
- (b) Show that the positive  $x$  and  $y$  axes are streamlines and, therefore, possible physical boundaries by showing that velocity components normal to the two coordinate axes vanish. [i.e. Show that  $u = \partial\phi(0, y)/\partial x = 0$  for  $0 < y < \infty$  and that  $v = \partial\phi(x, 0)/\partial y = 0$  for  $0 < x < \infty$ .]
- (c) Calculate the difference in piezometric heads between the points (1, 0) and (0, 2) metres.
- (d) Obtain the corresponding form of (6.22) that results when gravity is neglected. Then use this equation to calculate the difference in pressure between the points (1, 0) and (0, 2) if gravity is neglected.

6.3



Flow to a sink next to a wall can be generated by placing a sink at  $(-L, 0)$  and a second sink of equal strength at the image point  $(L, 0)$ .

- Write the potential function for this flow.
- Calculate the velocity vector function on the wall at the point  $(0, y)$ . Notice that the velocity component normal to the wall vanishes, which proves that the wall is a streamline.
- Assume that the pressure vanishes at infinity. Use the Bernoulli equation to calculate the pressure on the wall at  $(0, y)$ . Then integrate this pressure to obtain the force on the wall. Neglect gravity in this calculation.

6.4 Replace the streamline  $y = 0$  in Example 6.1 with the ground surface to model the flow of wind across the roof of a plastic covered tunnel house.

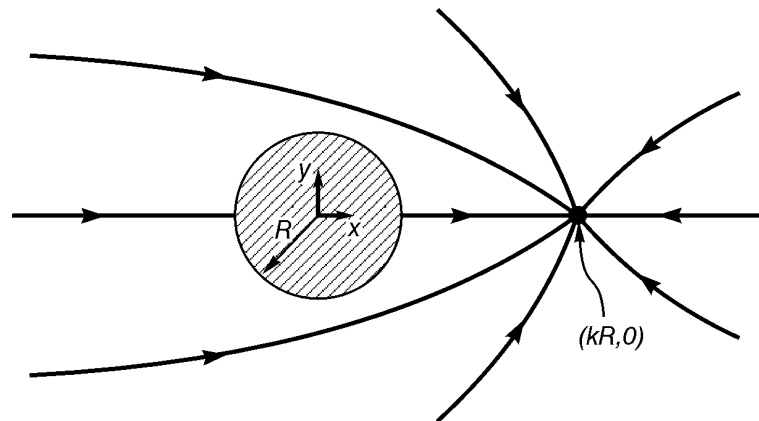
- Integrate the pressure over the entire roof to show that the upward lift force on the roof, per unit width, is

$$\mathbf{F} = \frac{5}{3} R \rho U_0^2 \mathbf{j}$$

- Suppose that air at  $10^\circ\text{C}$  blows across a 40 m long tunnel house with  $R = 10$  m. Assume that the roof and supporting structural elements are weightless. How many men weighing 80 kg each would be required to hold down the roof in a 40 km/hr wind?

The diverging streamlines on the leeward side of the roof suggest that flow separation is likely to occur – a topic that will be discussed in Chapter 8. Even then, however, a substantial lift force will be exerted on the roof.

6.5



Flow past a circle of radius  $R$  to a sink at  $(kR, 0)$  with  $k > 1$  can be generated by placing a sink of strength  $q$  at  $(kR, 0)$ , a sink of strength  $q$  within the circle at  $(R/k, 0)$  and a source of strength  $q$  within the circle at  $(0, 0)$ . (This result was calculated by using an elegant bit of complex variable theory known as the Milne-Thomson circle theorem.) The converging streamlines suggest that boundary layers are likely to be thin and separation is unlikely if Reynolds numbers are not too low.

- Use the potential functions for a source and sink to obtain the velocity potential for this flow in Cartesian coordinates.
- Rewrite the potential function in polar coordinates by setting  $x = r \cos \theta$  and  $y = r \sin \theta$ . Then prove that this potential solves the stated problem by showing that  $u_r = \partial \phi / \partial r = 0$  on the cylinder surface  $r = R$ .
- D'Alembert's paradox, which is mentioned in Example 6.1 and proved in Chapter 9, fails to hold when singularities exist at a finite distance from an object in irrotational flow. In this problem a force is created on the cylinder that is directed toward the sink at  $(kR, 0)$ . Calculate the velocity  $u_\theta = (1/r) \partial \phi / \partial \theta$  on the cylinder surface to obtain

$$u_\theta(R, \theta) = -\frac{q}{\pi R} \frac{k \sin \theta}{k^2 - 2k \cos \theta + 1}$$

Then neglect gravity and use this velocity with the Bernoulli equation to show that the force per unit width on the cylinder is

$$F = iR\rho \left( \frac{q}{\pi R} \right)^2 \int_0^\pi \frac{k^2 \sin^2 \theta \cos \theta}{(k^2 - 2k \cos \theta + 1)^2} d\theta$$

The integral is difficult to evaluate. However, one integration by parts and use of the known integral\*

$$\int_0^{\pi} \frac{\cos(n\theta)}{k^2 - 2k \cos \theta + 1} d\theta = \frac{\pi}{(k^2 - 1)k^n} \quad \text{for } k^2 > 1$$

leads to the final result

$$\mathbf{F} = \frac{\rho q^2}{2\pi R k(k^2 - 1)} \mathbf{i}$$

6.6 Show that the following expressions for  $\phi$  satisfy the Laplace equation, (6.15). Then calculate the corresponding stream function,  $\psi$ , by integrating Eqs. (6.45 a, b).

(a)  $\phi = U_0 x + A e^{ky} \cos(kx)$

(b)  $\phi = x^3 - 3xy^2 + y$

Finally, show that functions calculated for  $\Psi$  also satisfy the Laplace equation, (6.47).

6.7 Consider the flow net shown for irrotational flow over a fence in Fig. 6.12. Calculate and plot the dimensionless ground pressure upstream from the fence

$$\frac{P_b - P_a}{\rho V_a^2 / 2} = 1 - \left( \frac{V_b}{V_a} \right)^2$$

by using the flow net geometry and Eq. (6.68). Let point  $a$  be on the ground surface midway between the third and fourth equi-potential contours upstream from the fence, and let point  $b$  be on the ground surface midway between the remaining equi-potential contours. One point on this plot can also be obtained at the point of stagnation, where

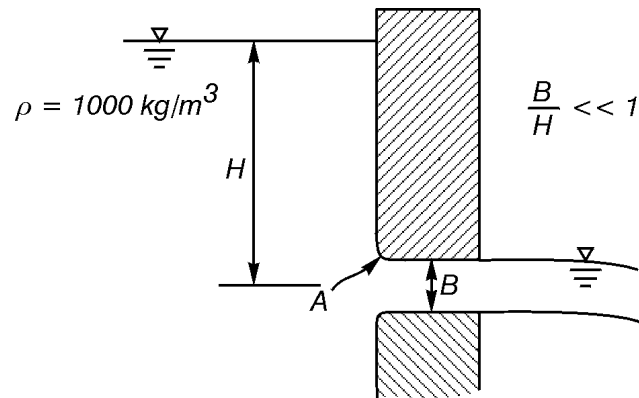
$$\frac{P_b - P_a}{\rho V_a^2 / 2} = 1$$

This latter result cannot be calculated from Eq. (6.68) but is obtained from Eq. (6.78).

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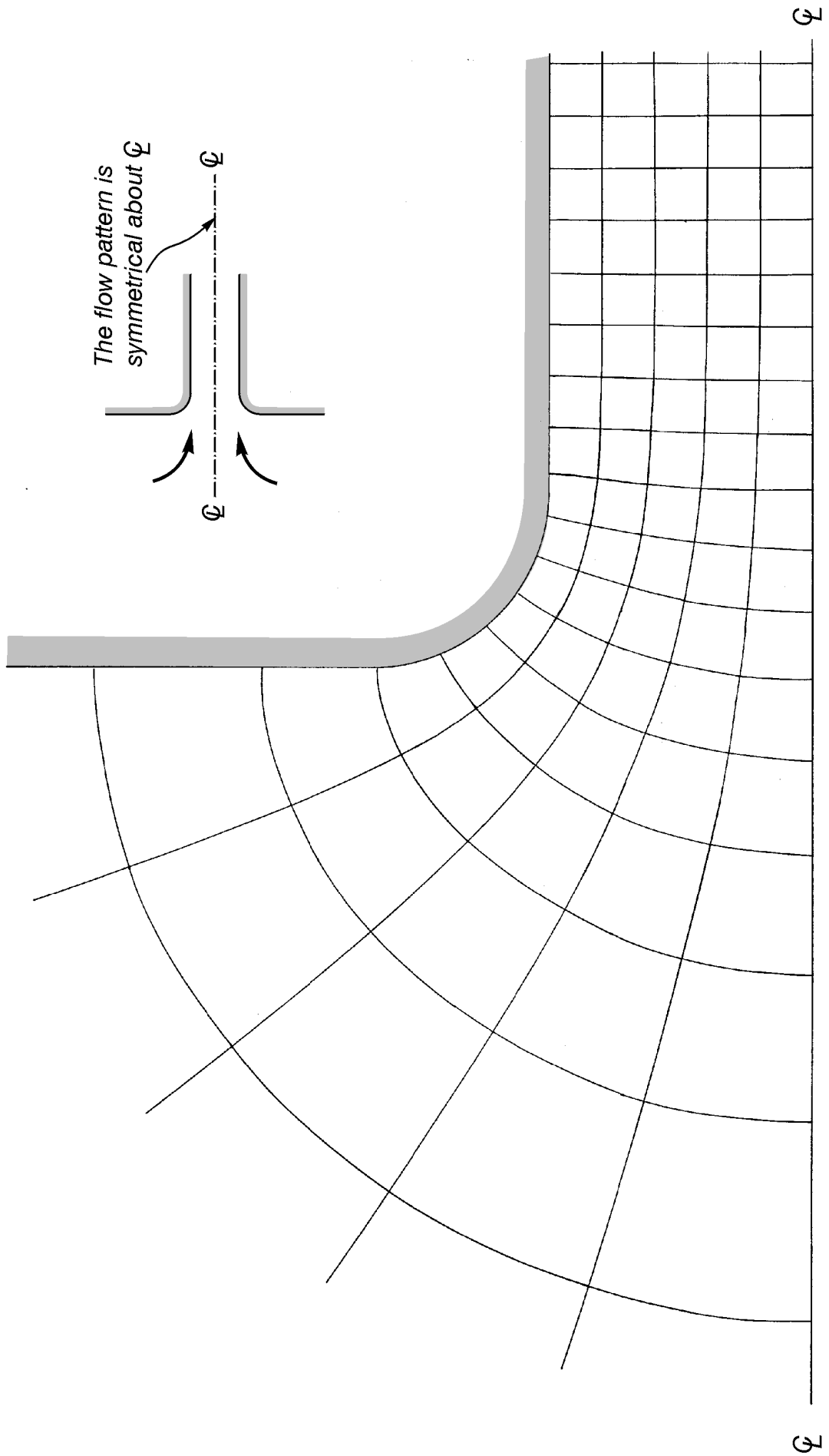
\* Gradshteyn, I.S. and I.M. Ryzhik (1965) *Table of Integrals, Series and Products*, Academic Press, New York, p. 366.

6.8



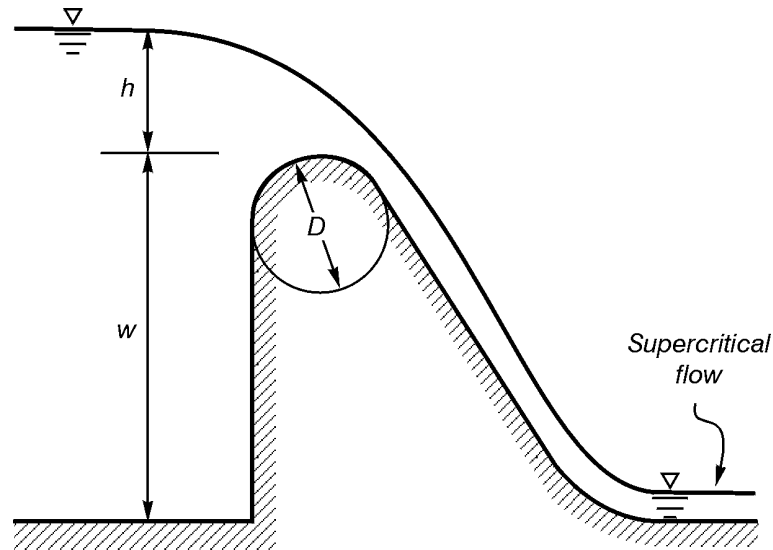
Water exits from a two-dimensional slot in a reservoir. Absolute zero pressure at sea level is  $-10.33$  m of water, and the vaporization pressure of water at  $10^\circ\text{C}$  is  $0.12$  m of water above absolute zero. Thus, cavitation will occur at the first point along a boundary where the gage pressure reaches  $p = -10.33 + 0.12 = -10.21$  m of water. Since  $B/H \ll 1$ , the velocity in the free jet is  $\sqrt{2gH}$ . Use these facts and the flow net on the following page to calculate the value of  $H$  which is just sufficient to cause cavitation on the boundary at point  $A$ .

Hint: Cavitation will occur at the point along the boundary where  $V$  is a maximum. This is also the point where  $\Delta s$  is a minimum.



Flow Net for Problem 6.8

6.9



Cassidy\* used irrotational flow theory to calculate the following discharge coefficients for flow over a spillway:

| $h/D$ | $C_D$ |
|-------|-------|
| 1     | 0.715 |
| 1.5   | 0.810 |
| 2.0   | 0.875 |
| 2.5   | 0.920 |
| 3.0   | 0.937 |

These calculations had  $w/D = 3.0$  and defined  $C_D$  by

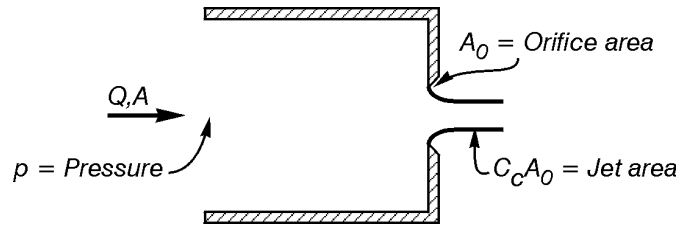
$$q = \frac{2}{3} C_D \sqrt{2gh^3}$$

in which  $q =$  flow per unit width. Calculate the supercritical flow depth ( $V^2/\sqrt{gy} = q^2/\sqrt{gy^3} < 1$ ) at the spillway base and the resultant horizontal force per unit width upon the spillway if  $w = 5$  m and  $h = 2$  m. Then compute the ratio of this force to the hydrostatic pressure force upon a portion of the dam that extends above the reservoir free surface. Explain why you would expect this ratio to be less than unity.

\* Cassidy, J.J. (1965) "Irrotational flow over spillways of finite height", *Jnl Engrg Mech. Div.*, ASCE, Dec., pp. 155-173.

6.10 An open channel carries a two-dimensional flow rate of  $q = 0.5 \text{ m}^2/\text{s}$ . A vertical sluice gate is to be used to maintain a depth of 2 m upstream from the gate. What is the required gate opening,  $b$  ?

6.11



Neglect gravity to show that the discharge coefficient in the definition

$$Q = C_D A_0 \sqrt{p/\rho}$$

is given for either a slot or circular orifice by

$$C_D = C_C \sqrt{\frac{2}{1 - (C_c A_0/A)^2}}$$

in which  $C_c$  = contraction coefficient = function of  $A_0/A$ . Calculate  $C_D$  for both a slot and a circular orifice when  $A_0/A = 0.50$ .

6.12 Give one answer for each of the following multiple choice questions.

- 1 Which of the following functions is not a potential function for irrotational flow?
  - (1)  $x$
  - (2)  $y$
  - (3)  $x^2 - y^2$
  - (4)  $\sin x$
  - (5)  $xy$
  
- 2 What is the  $x$  velocity component calculated from the potential function  $\phi = x^3 - 3xy^2$ ?
  - (1)  $-6xy$
  - (2)  $3(x^2 - y^2)$
  - (3)  $-3y^2$
  - (4)  $x^4/4 - 3x^2y^2/2$
  - (5) none of the choices given

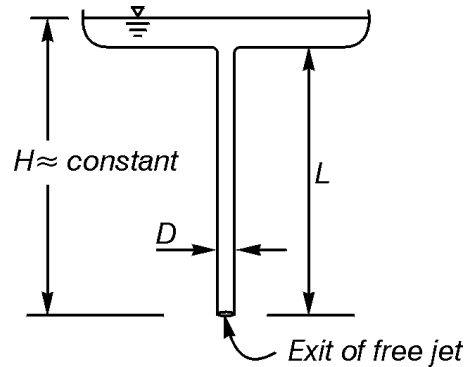
- 3 What is the stream function,  $\Psi$ , for the irrotational flow that has the velocity potential function  $\phi = x^3 - 3xy^2$ ?
- (1)  $3x^2y - y^3 + k$
  - (2)  $3x^2y + k$
  - (3)  $x^4/4 - 3x^2y^2/2 + k$
  - (4)  $x^3y - xy^3 + k$
  - (5) none of the choices given
- 4 Irrotational flow past a circular cylinder that is uniform at infinity and has a non-zero circulation is obtained by adding the potential for uniform flow to the potential or potentials for
- (1) a source
  - (2) a source and a sink
  - (3) a vortex and a source
  - (4) a vortex and a doublet
  - (5) a vortex and a sink
- 5 Which of the following choices is not characteristic of irrotational flow around a bend between two fixed, curved boundaries?
- (1) Velocities increase toward the centre of curvature.
  - (2) The velocity is a minimum on the outside boundary of the bend.
  - (3) The pressure is a minimum on the inside boundary of the bend.
  - (4) The fluid slips along the inside boundary of the bend.
  - (5) The velocity magnitude is zero on the outside boundary of the bend.
- 6 Which of the following choices is not a characteristic of flow net solutions?
- (1) A flow net is constructed of curvilinear squares.
  - (2) Velocity magnitudes decrease as the size of the flow net elements increase.
  - (3) Velocity ratios can be calculated only at mid-points of curvilinear squares.
  - (4) Any streamline can be replaced with a fixed physical boundary.
  - (5) Flow net construction proceeds by using a method of successive approximation.

- 7 Which of the following is not true for high Reynolds number flow past a sharp corner on a physical boundary?
- (1) A viscous flow separates at the sharp corner if the interior angle is greater than  $180^\circ$ .
  - (2) The viscous and irrotational flow velocities are both zero if the interior angle is less than  $180^\circ$ .
  - (3) Streamlines and potential lines meet at right angles at a sharp corner in irrotational flow.
  - (4) Pressure reaches a maximum at a sharp corner if the interior angle is less than  $180^\circ$ .
  - (5) A flow net cannot be used to calculate the velocity at a sharp corner.
- 8 Which of the following is not true for flow next to a physical boundary.
- (1) Irrotational flow slips along the boundary.
  - (2) The normal velocity component vanishes for both viscous and irrotational flow.
  - (3) A viscous flow has a zero tangential velocity component along the boundary.
  - (4) Pressures are calculated from irrotational flow approximations when Reynolds numbers are large.
  - (5) Irrotational flow has a zero tangential velocity component along the boundary.
- 9 A free streamline in steady irrotational flow requires for boundary conditions
- (1) zero normal and tangential velocity components
  - (2) zero normal velocity component and zero pressure
  - (3) zero circulation
  - (4) zero velocity
  - (5) zero tangential velocity component and zero pressure
- 10 Which of the following types of flow at high Reynolds numbers would not be closely approximated with irrotational flow?
- (1) Flow accelerated rapidly from a state of rest.
  - (2) Flow in which streamlines converge rapidly.
  - (3) Flow with relatively thin boundary layers.
  - (4) Flow in which fluid particles are rapidly decelerated as they move with the flow.
  - (5) Flow accelerated rapidly from a state of motion that is initially irrotational.

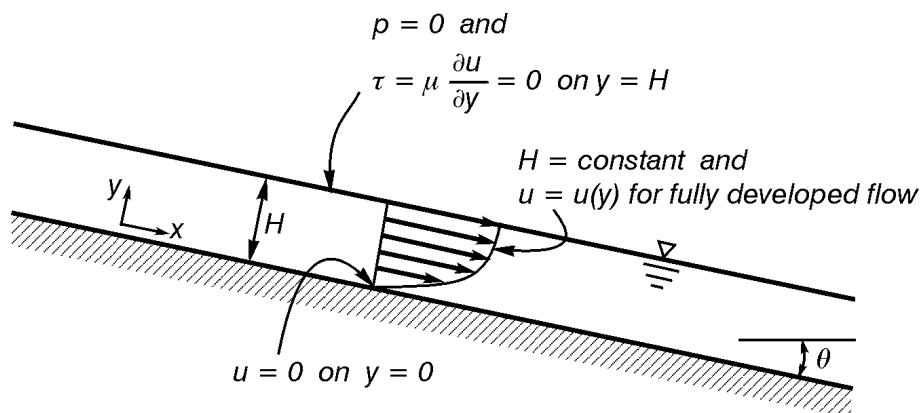
# CHAPTER 7

## LAMINAR AND TURBULENT FLOW PROBLEMS

- 7.1 A viscometer consists of a small diameter tube in the bottom of a reservoir. Assume that the reservoir level remains constant, that the flow in the tube is fully developed laminar flow and that the free jet does not contract at its exit. Calculate the fluid viscosity in terms of the measured flow rate,  $Q$ , and the viscometer geometry.



- 7.2 Debris flows are mixtures of water, soil and rock that flow down steep hillsides when highly saturated soil becomes unstable. It is generally agreed that the front portion of these flows remain laminar, although not everyone believes that they can be modelled as Newtonian flows. A sketch for fully developed laminar flow is shown below:



- (a) Show that the definition of  $h$  in Eq. (2.22) give the following result for this flow:

$$h = \frac{p}{\rho g} - x \sin \theta + y \cos \theta$$

- (b) Since  $v = 0$  everywhere, Eq. (2.28 b) reduces to

$$\frac{\partial h}{\partial y} = 0$$

Since this equation shows that  $h$  does not change along any line parallel to the  $y$  axis, use the definition of  $h$  in part (a) to show that  $h$  has the following value throughout the flow:

$$h = -x \sin \theta + H \cos \theta$$

Thus, equating this value of  $h$  with the value of  $h$  in part (a) shows that pressures are given by

$$p = \rho g (H - y) \cos \theta$$

- (c) Use the expression for  $h$  in part (b) to show that Eq. (2.28 a) reduces to

$$g \sin \theta + \nu \frac{d^2 u(y)}{dy^2} = 0$$

Then integrate this equation and apply appropriate boundary conditions to show that

$$u(y) = \left( yH - \frac{y^2}{2} \right) g \sin \theta / \nu$$

- (d) Calculate the flow rate per unit width

$$q = \int_0^H u(y) dy$$

and the bed shear  $\tau_0 = \mu \partial u / \partial y$  on  $y = 0$ .

- (e) Comparisons with field data made by Hunt (1994)\* suggest that a typical viscosity for a debris flow is  $\nu \approx 1 \text{ m}^2/\text{s}$ . Calculate the maximum velocity  $[u(H)]$ , the flux velocity ( $q/H$ ) and the Reynolds number ( $Re = VL/\nu$  with  $V = q/H$  and  $L = H$ ) for a flow which has  $H = 2\text{m}$  and  $\theta = 30^\circ$ .

- 7.3 Figure 7.3 shows that

$$\frac{U_\infty D}{\nu} = 1$$

is the largest Reynolds number for which Eq. (7.18) can be used to predict a particle terminal velocity. Calculate the corresponding values for  $U_\infty$  and  $D$  for sand ( $\rho_s/\rho_f = 2.65$ ) in fresh water at  $10^\circ\text{C}$ . These would be upper bounds for  $U_\infty$  and  $D$  since increasing either of these two variables beyond these values would also increase  $Re$  beyond unity.

- 7.4 A measured value for a permeability in a vertical Hele-Shaw experiment using oil is 20 mm/s for a plate spacing of 2.54 mm. What is the viscosity of the oil?
- 7.5 A 1:30 scale Hele-Shaw model of flow through an embankment gives  $q = 0.0005 \text{ m}^2/\text{s}$  and has a permeability of  $K = 0.015 \text{ m/s}$ . Calculate the corresponding value of  $q$  for a prototype that has  $K = 0.003 \text{ m/s}$ .

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\* Hunt, B. (1994) "Newtonian fluid mechanics treatment of debris flows and avalanches", *Jnl Hyd. Engrg*, ASCE, Vol. 120, No. 12, pp. 1350-1363.

7.6 In a careful analysis of flow instability and the transition to turbulence, a distinction must be made between the *point of instability*, which is the first point in the flow where a disturbance becomes amplified, and the *point of transition*, which is the point where turbulence extends throughout a major portion of the flow. Since it must take a finite time for an instability to spread throughout a larger region as it is transported downstream, we would expect the point of transition to lie downstream from the point of instability. Use the following calculations to verify this for a laminar boundary layer on a flat plate:

- (a) The point of instability, which is shown with Fig. 7.12, is the smallest Reynolds number at which any disturbance can be amplified. This is usually taken to be

$$\frac{U_{\infty} \delta_1}{\nu} = 420$$

for a laminar boundary layer on a flat plate. The boundary layer displacement thickness,  $\delta_1$ , is about one third of the boundary layer thickness,  $\delta$ , for a laminar boundary layer. Furthermore, the Blasius solution for a laminar boundary layer is

$$\frac{\delta}{x} = \frac{5}{\sqrt{U_{\infty} x / \nu}}$$

Use these two facts to show that the point of instability has an  $x$  coordinate given by

$$\frac{U_{\infty} x_i}{\nu} = 6.35 \times 10^4$$

- (b) The point of transition is usually taken to have an average  $x$  coordinate given by

$$\frac{U_{\infty} x_T}{\nu} = 5 \times 10^5$$

for a laminar boundary layer on a flat plate, which is about ten times larger than the  $x$  coordinate of the point of instability. (This point of transition is shown in Figs. 8.4 and 8.5.

Schlichting (1968)\* gives a detailed discussion of this idea and shows an experimental plot that suggests that  $x_i$  and  $x_T$  approach each other as the turbulence intensity of the flow outside the laminar boundary layer increases.

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\* Schlichting, H. (1968) *Boundary-layer Theory*, 6th edition, McGraw-Hill Book Co., New York, pp. 455-463.

7.7 Water at  $5^{\circ}\text{C}$  flows in a commercial steel pipe that has a diameter of 100 mm. Use Fig. 7.18 to calculate  $f$  for the following velocities:

- (a) 10 mm/s
- (b) 100 mm/s
- (c) 2 m/s

In each case, state whether the flow is laminar, turbulent through a smooth pipe or turbulent through a rough pipe.

7.8 How many nozzle diameters downstream must one go in an axisymmetric jet to reach the points where the maximum velocity and maximum concentration have decayed to values that are one per cent of their values at the nozzle exit? Repeat the calculation for two-dimensional flow from a slot. Although Eqs. (7.57) and (7.62) appear to give very definitive answers for this problem, a word of caution is in order. These are time-averaged concentrations. In Fischer, List and Koh (1979) it is pointed out that the maximum centre line concentration for a two-dimensional flow exceeds the time-averaged concentration by almost 70 per cent.

## CHAPTER 8

### BOUNDARY-LAYER FLOW PROBLEMS

- 8.1 The accuracy of approximation when using the Pohlhausen technique to solve Eq. (8.6) depends upon the form of the function that is used to approximate the velocity distribution within the boundary layer. For example, replacement of the second degree polynomial used in Example 8.1 with a third or fourth degree polynomial gives a more accurate approximation to the exact solution of the partial differential equations. (Then the governing partial differential equations must be used to determine additional constants in the higher degree polynomial.) A less accurate solution can be obtained by using a first degree polynomial:

$$u(x, y) = a(x) + b(x)y$$

Since only two coefficients must be determined, one of the three boundary conditions used in example 8.1 must be dropped. Obtain a solution for  $\delta/x$  and  $C_D$  with this first degree polynomial by using the following two boundary conditions to determine  $a(x)$  and  $b(x)$ :

$$u(x, 0) = 0 \quad \text{and} \quad u(x, \delta) = U$$

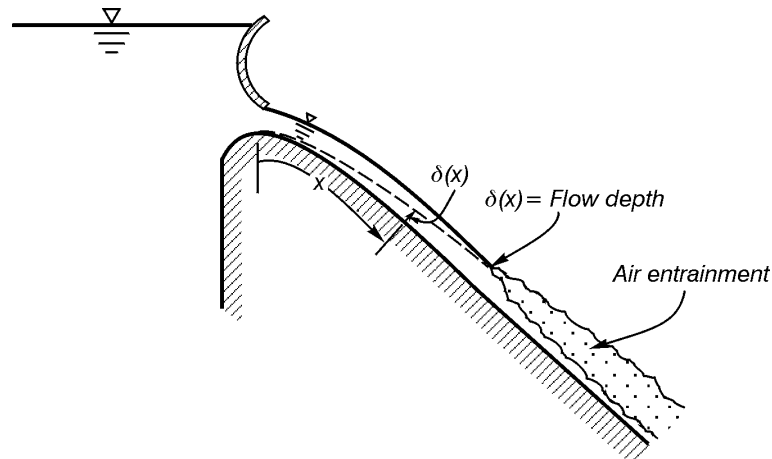
in which  $U = \text{constant velocity at } y = \delta$ . Then compare the error of approximation with the error for the second degree polynomial used in Example 8.1.

- 8.2 The function

$$u(x, y) = U \sin(\xi\pi/2) \quad \text{in which } \xi = y/\delta(x)$$

satisfies the same three boundary conditions that were used for the second degree polynomial in Example 8.1. Use this expression for  $u$  to obtain  $\delta/x$  and  $C_D$  for a laminar boundary layer on a flat plate. Then calculate the errors of approximation.

8.3



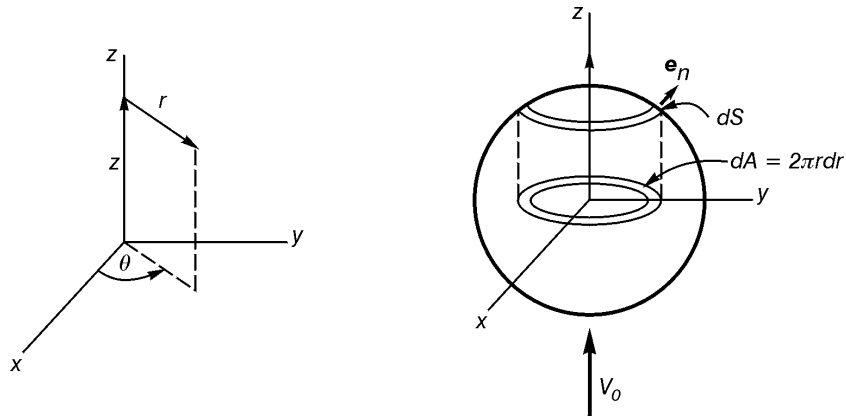
The lateral mixing created by turbulence is capable, at large enough Reynolds numbers, of mixing air into water across an air-water interface (air entrainment). Air entrainment occurs on spillways when a turbulent boundary layer, which starts its development on the spillway crest, thickens sufficiently to intersect the free surface. Downstream from this point the flow becomes frothy and milky white, and flow depths increase with distance downstream in a phenomenon known as "bulking". Engineers are concerned with bulking because a spillway design must include enough freeboard along the spillway sides to prevent overtopping. Use the expression for a turbulent boundary layer thickness calculated in Example 8.2 from the one seventh power law to estimate the distance downstream from a sluice gate at which air entrainment starts for a flow at  $10^\circ\text{C}$  that has an average velocity and depth of 10 m/s and 200 mm, respectively.

- 8.4 Calculate the drag force on a 100 m length of 25 mm diameter cable that is towed through fresh water at  $10^\circ\text{C}$  with a speed of 20 km/hr. Assume that flow velocities are parallel to the cable, that the cable has a roughness height of  $\epsilon = 2$  mm and that the results for flow along a flat plate can be used to estimate the drag force. Convert this force in Newtons to the equivalent mass at sea level by dividing the force by  $g = 9.81$  m/s<sup>2</sup>.
- 8.5 Use the results of Example 8.4 to estimate the increase in depth on the downwind side of a reservoir that has a fetch of 1 km, a depth of 2 m, a wind speed of 30 m/s and wave heights of  $\epsilon = 0.8$  m. Assume that the fresh water and air both have a temperature of  $10^\circ\text{C}$ , and note that the required increase in water depth is  $(h_2 - h_1)/2$  if the free surface tilts as a plane. How far downwind will the reservoir edge move in the horizontal direction if the ground has a slope of 1:1000?

# CHAPTER 9

## DRAG AND LIFT PROBLEMS

- 9.1 Consider the pressure distribution shown for a sphere in Fig. 9.2 (b). The value of  $C_D$  for this case is 0.5. Carry out the following calculations to see what portion of the total drag force is contributed by the constant negative wake pressure.



The force due to pressure acting on any portion,  $S$ , of the sphere surface is

$$\mathbf{F} = \int_S \Delta p (-\mathbf{e}_n) dS$$

in which  $\Delta p$  = change in pressure from its value at infinity and  $\mathbf{e}_n$  = outward normal. Calculate the component of  $\mathbf{F}$  in the direction of the drag force by dotting  $\mathbf{F}$  with  $\mathbf{k}$  to obtain

$$\mathbf{k} \cdot \mathbf{F} = - \int_S \Delta p (\mathbf{k} \cdot \mathbf{e}_n) dS$$

Since  $(\mathbf{k} \cdot \mathbf{e}_n) dS = \cos \theta dS = dA = 2\pi r dr =$  projection of  $S$  upon the  $xy$  plane, this becomes

$$\mathbf{k} \cdot \mathbf{F} = - \int_0^{r_0} \Delta p 2\pi r dr$$

in which  $r_0$  = maximum radius of the projection of  $S$  upon the  $xy$  plane. Since  $\Delta p$  is constant on  $S$ , we can write

$$\Delta p = -\epsilon \rho \frac{V_0^2}{2}$$

in which  $\epsilon$  must be scaled from Fig. 9.2 (b). Thus, the contribution of the constant negative wake pressure to the total drag is

$$\mathbf{k} \cdot \mathbf{F} = \epsilon (\pi r_0^2) \rho \frac{V_0^2}{2}$$

What portion of the total drag is this value?

- 9.2 A smooth square flat plate with sides of length 0.5 m is submerged in water at 10°C that has an undisturbed flow velocity of 2 m/s. Calculate the force on the plate (a) when it is aligned with the flow and (b) when it is normal to the flow. Then calculate the ratio of these two forces.
93. Use the expression derived in Example 9.1 to calculate the terminal fall velocity of a 1 mm diameter sand particle (specific gravity = 2.65) in water at 10°C. Then use the expression derived in Example 9.2 to estimate the number of diameters and the dimensional distance that the particle falls when released from rest before reaching 95 per cent of its terminal fall velocity.
- 9.4 The wind drag on a building in a 40 km/hr wind at 20°C is to be found by using a 1:300 scale model in a flume that contains water at 10°C. Calculate the prototype to model force ratio if the water has a velocity of 0.05 m/s. Assume that the building has an angular geometry with sharp corners so that Reynolds number and relative roughness effects can be neglected.
- 9.5 The rate of fuel consumption required to overcome a force is proportional to power, which is the dot product of the force and velocity vectors. Thus, the amount of fuel consumed in travelling along a path between two fixed points equals the time integral of the fuel consumption rate, and this turns out to be proportional to the work done by the force along the path.

$$\text{Fuel consumption rate} \propto \text{power} = \mathbf{F} \cdot \mathbf{V}$$

$$\therefore \text{Fuel consumed} \propto \int_{t_a}^{t_b} \mathbf{F} \cdot \mathbf{V} dt = \int_a^b \mathbf{F} \cdot d\mathbf{r} = \text{Work}$$

Therefore, if the drag coefficient is constant, the fuel consumption rate for a car to overcome wind drag is proportional to  $V^3$ , but the amount of fuel required to overcome wind drag in travelling between two specified points is proportional to  $V^2$ . Assume that the Volkswagen van in Example 9.4 has a mass of 1500 kg, a projected area of 5 m<sup>2</sup> and  $C_D = 0.42$ . Also assume that drag and gravity are the two primary forces that do work on the van. Calculate the percentage increase in volume of fuel consumed for the van to go 10 km along (a) a level road, (b) a road that slopes upward one degree and (c) a road that slopes upward at five degrees if the speed is increased from 80 to 100 km/hr. Use an air temperature of 10°C.

- 9.6 It was suggested in problem 7.2 that velocities in a debris flow might be approximated with

$$u(y) = \left( yH - y^2/2 \right) g \sin \theta / \nu$$

Follow the procedure used in Example 9.5 to estimate the drag force on a bridge pier in a debris flow if the bridge pier has the shape of a circular cylinder of radius  $R$ . Assume that the flow depth,  $H$ , is less than the bridge pier height.

- 9.7 Choose and sketch an aerofoil with any reasonable angle of attack. Then sketch a flow net and use Eq. (9.10) and the method shown by Eqs (9.11) - (9.15) to calculate a lift coefficient for the aerofoil.

9.8 Vibrations in a tightly stretched cable, wire or rope caused by vortex shedding are described by the following equations:

$$c^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = \frac{F_0}{m} \sin(\omega t) \quad \text{for } 0 < x < \ell$$

$$u(0, t) = 0$$

$$u(\ell, t) = 0$$

(a) Show by direct substitution that the following function is a solution of the above equations:

$$u(x, t) = \frac{F_0}{m\omega^2} \left[ \frac{\sin(\omega\ell/c) - \sin(\omega x/c) - \sin \omega(\ell - x)/c}{\sin(\omega\ell/c)} \right] \sin(\omega t)$$

(b) The maximum magnitude of  $u$  occurs at  $x = \ell/2$  when  $\sin(\omega t) = 1$ . Thus, the maximum magnitude of  $u$  can be written in the following dimensionless form:

$$\frac{\text{Max}|u|}{F_0/(m\omega^2)} = \left| \frac{\sin(\theta) - 2\sin(\theta/2)}{\sin(\theta)} \right| = \left| \frac{1}{\cos(\theta/2)} - 1 \right|$$

in which  $\theta = \omega\ell/c = 2\pi(\ell/D)(U_\infty/c)S$ . Plot  $\text{Max}|u|/(F_0/m\omega)$  versus  $\theta$  for  $0 \leq \theta \leq 6\pi$ . This plot shows that  $\text{Max}|u|$  is a continuous function of the dimensionless frequency  $\theta$  and that  $\theta$  only has to be sufficiently close to a resonant frequency at  $\theta = (2n - 1)\pi$  to create large magnitudes for  $u$ .



# CHAPTER 10

## DIMENSIONAL ANALYSIS AND MODEL SIMILITUDE PROBLEMS

10.1 Surface tension,  $\sigma$ , has units of force per unit length (N/m). Since Newton's second law states that  $N = ML/T^2$ , show that the basic dimensions of  $\sigma$  in the  $MLT$  system are  $MT^2$ .

10.2 Molecular diffusion in laminar flow is described by solutions of the diffusion equation:

$$D \frac{\partial^2 c}{\partial x^2} = u \frac{\partial c}{\partial x} + \frac{\partial c}{\partial t}$$

in which  $x$  = distance (m),  $t$  = time (s),  $u$  = velocity (m/s) and  $c$  = concentration (mg/litre). Use the fact that this equation is dimensionally correct (i.e., the dimensions of each term are identical) to determine the dimensions of the diffusion coefficient,  $D$ .

10.3 The pressure gradient,  $dp/dx$ , for laminar flow in a pipe is known to be a function of the following variables:

$$\frac{dp}{dx} = f(D, V, \mu)$$

in which  $D$  = pipe diameter,  $V$  = mean velocity and  $\mu$  = dynamic viscosity. Use the fact that a Reynolds number  $VD\rho/\mu$  is dimensionless to determine the dimensions of  $\mu$ . Then carry out a dimensional analysis to show that the following variable is the only dimensionless variable that results:

$$\frac{D^2 dp/dx}{V\mu}$$

Since there are no other dimensionless variables, this result shows that

$$\frac{D^2 dp/dx}{V\mu} = \text{constant}$$

The numerical magnitude of this dimensionless constant is 32, but dimensional analysis cannot determine this number. Only mathematics or experiment can do this.

10.4 A drag force of 15 N is measured on an object in a wind tunnel for a wind speed of 10 m/s. Calculate the corresponding drag force and wind speed for the prototype if the scale ratio is 1:20 and if air temperatures are identical for model and prototype.

- 10.5 Now assume that the points of flow separation around the object in problem 10.4 are fixed at sharp corners so that they do not change with Reynolds number. Also assume that surface drag is relatively small. Then Eq. (10.18) reduces to

$$\frac{F/A}{\rho U_{\infty}^2/2} = \text{constant}$$

Calculate the force on the prototype for a wind speed of 30 m/s.

- 10.6 A simply supported beam has a weight,  $W$ , suspended from its midpoint. Deflections along the beam are calculated by solving the following problem:

$$\begin{aligned} EI \frac{d^2y}{dx^2} &= \text{Moment} = Wx/2 \quad \text{for } 0 < x < \ell/2 \\ &= Wx/2 - W(x - \ell/2) \quad \text{for } \ell/2 < x < \ell \\ y(0) &= 0 \\ y(\ell) &= 0 \end{aligned}$$

Thus, the deflection is seen to be a function of the following variables:

$$y = f\left(x, \ell, \frac{W}{EI}\right)$$

Use a dimensional analysis to show how you would plot the results (calculated or experimental) dimensionlessly.

- 10.7 The flow rate seeping through an earth embankment is a function of the following variables:

$$q = f(\ell, K, H)$$

in which  $q$  = flow rate per unit width,  $\ell$  = base width of the embankment,  $K$  = coefficient of permeability (m/s) and  $H$  = reservoir depth. Use a dimensional analysis to show how you would plot results from either a model study or calculations so that they could be used for all geometrically similar embankments.

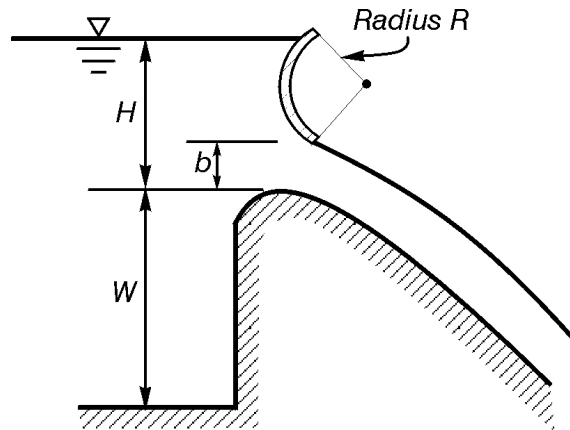
- 10.8 A 1:30 scale model of a spillway measures a flow rate per unit width,  $q$ , of 0.05 m<sup>2</sup>/s for a head,  $H$ , of 0.1 m. Assume that flow depths and velocities are large enough to allow Reynolds number and relative roughness effects to be neglected. Follow the procedure used in Examples 10.1 - 10.2 to calculate corresponding values of  $q$  and  $H$  for the prototype.

- 10.9 The pressure distribution along the floor of a stilling basin is a function of the following variables:

$$p = f(\ell, x, \rho, U, g, H)$$

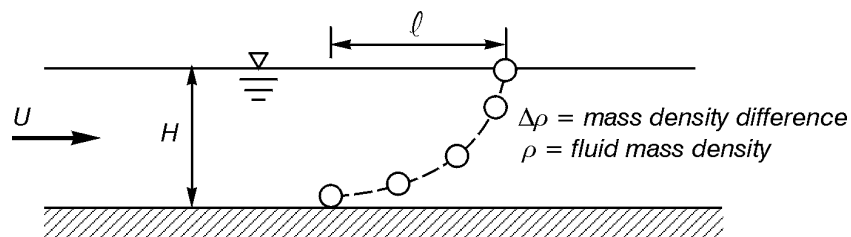
in which  $p$  = pressure,  $\ell$  = stilling basin length,  $x$  = coordinate of the point where  $p$  is measured,  $\rho$  = fluid mass density,  $U$  = approach velocity,  $g$  = gravitational constant and  $H$  = flow depth. Carry out a dimensional analysis, putting the final results in the form of standardized dimensionless variables where possible.

- 10.10



The horizontal force per unit width,  $F$ , on a curved sluice gate on a spillway crest is to be studied experimentally. Use a dimensional analysis to show how you would plot the results in a general way. Then assume that you want to study the flow rate per unit width,  $q$ , and repeat the exercise.

- 10.11

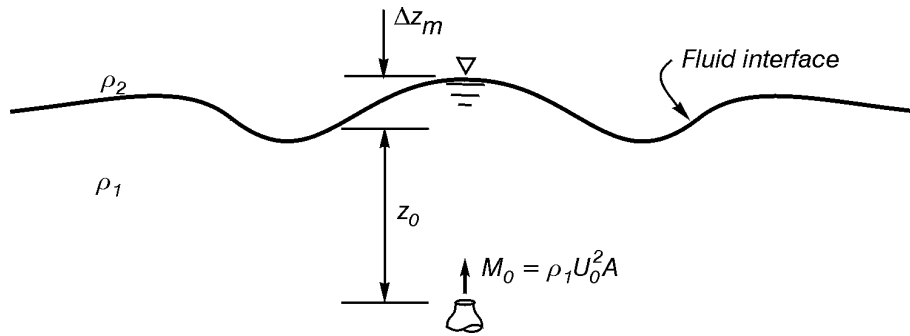


The trajectory of a buoyant particle released on the bottom of an open channel is to be used as a flow meter to measure the flux (average) velocity,  $U$ . The variables are

$$U = f\left(H, \ell, g, \frac{\Delta\rho}{\rho}\right)$$

Show how a dimensional analysis would allow you to plot the calibration data in a general way.

10.12



An axisymmetric jet is directed upward at a fluid interface to study the maximum interface deflection,  $\Delta z_m$ . Show how you would plot the experimental results if the nozzle is submerged deeply enough to act as a point source of momentum. Assume large Reynolds numbers with highly turbulent flow.

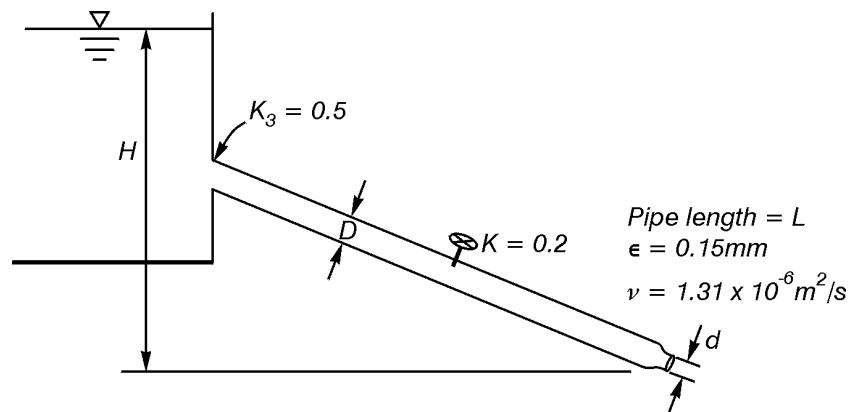
# CHAPTER 11

## STEADY PIPE FLOW PROBLEMS

11.1 Use Fig. 7.18 to determine  $f$  for the following flows:

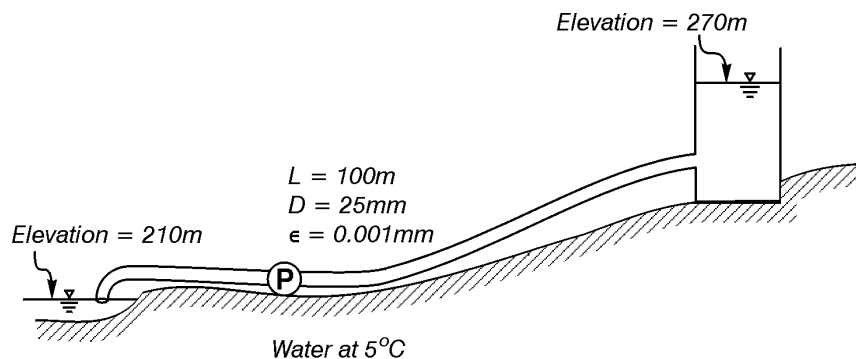
- (a)  $Re = 1 \times 10^6$  and  $\epsilon/D = 0.001$
- (b)  $Re = 1.5 \times 10^3$  and  $\epsilon/D = 0.001$
- (c)  $U = 5$  m/s,  $D = 100$  mm for water at  $10^\circ\text{C}$  in commercial steel pipe.

11.2



Sketch the hydraulic and energy grade lines. Then write an algebraic equation that could be solved for the flow rate in the pipe.

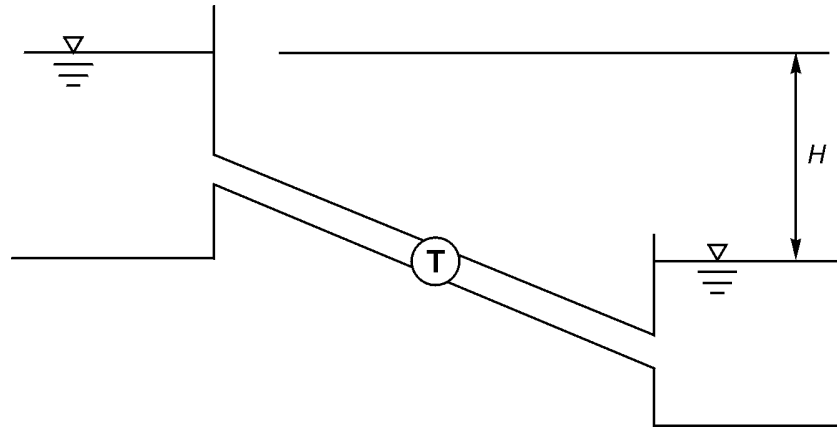
11.3 Calculate the flow rate in problem 11.2 if  $H = 50$  m,  $L = 200$  m,  $D = 100$  mm and  $d = 50$  mm. Then calculate the ratio of minor to friction loss terms.



11.4 Sketch the hydraulic and energy grade lines. Then calculate the power required to drive the pump if the pump efficiency is 85 per cent, if the flow velocity in the pipe is 3 m/s and if local losses are negligible. Finally calculate the input power for a pump motor that is 90 per cent efficient.

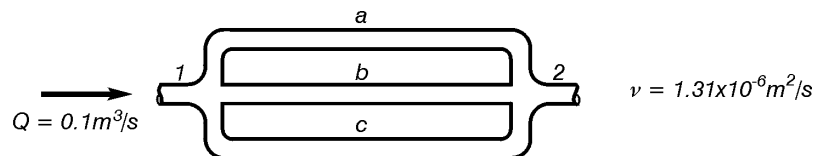
11.5 Rework problem 11.4 by including local losses at the pipe inlet and outlet. The inlet loss coefficient is given by Table 11.1, and the outlet loss coefficient is given by Eq. (11.6 b) with  $A_2 = \infty$ . Calculate the percentage differences between your answers and the answers obtained by neglecting local losses.

11.6



A pump is used in reverse as a turbine and has an efficiency of 60 per cent as water runs under gravity to the lower reservoir. Sketch the hydraulic and energy grade lines. Then write two algebraic equations that could be solved simultaneously with a characteristic curve for the "turbine" to determine the flow rate, the change in head across the turbine and the power delivered by the "turbine" to the drive shaft. Include local loss terms for the inlet and outlet.

11.7



| Pipe | $L$  | $D$    | $f$   |
|------|------|--------|-------|
| a    | 60 m | 100 mm | 0.02  |
| b    | 50 m | 75 mm  | 0.01  |
| c    | 60 m | 50 mm  | 0.015 |

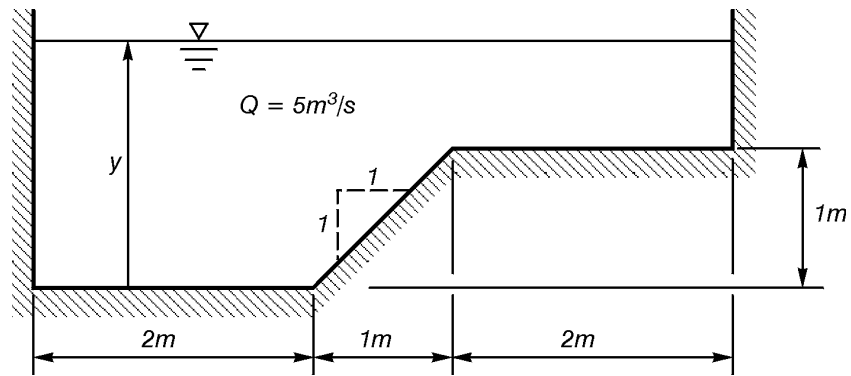
Use Eqs. (11.13) - (11.15) at nodes 1 and 2 to obtain two equations with the heads at these two nodes as the only unknowns. Show that these two equations are identical so that  $h_1$  and  $h_2$  are not determined uniquely from these two equations. Then obtain from either one of these equations the unique solution for  $\Delta h = h_1 - h_2$  and for the flow rate in each pipe.

## CHAPTER 12

### STEADY OPEN CHANNEL FLOW PROBLEMS

- 12.1 A rectangular open channel has a width of 8 m and a flow of  $5 \text{ m}^3/\text{s}$ . Calculate the downstream depth and gate opening height if a sluice gate is used to maintain an upstream depth of 1 m.
- 12.2 A rectangular open channel has a width of 8 m and a flow of  $5 \text{ m}^3/\text{s}$ . Calculate the downstream depth and gate opening height if a sluice gate is used to maintain an upstream depth of 2 m.
- 12.3 Calculate the upstream and downstream depths in problem 12.1 if the gate opening height is 0.2 m.
- 12.4 A rectangular channel has a width of 3 m and a flow of  $4 \text{ m}^3/\text{s}$ . Calculate the height,  $\Delta z$ , of a hump that can be placed in the channel to cause a hydraulic jump to stabilize immediately downstream if the normal depth that would occur with no hump present is calculated from Eq. (12.34) as 1.5 m. What is the depth immediately upstream from the hump? (Assume that normal depth occurs on the downstream side of the jump. You will have to change slightly the equations derived in Example 4.7 to calculate the depth on the upstream side of the jump.)
- 12.5 Explain with sketches and words what would happen as the downstream water depth in problem 12.3 is raised gradually to the depth just upstream from the hump. Then explain what would happen if the downstream depth is increased still more.
- 12.6 A rectangular channel has a width of 4 m, a flow depth of 2 m and a flow of  $6 \text{ m}^3/\text{s}$ . The channel width narrows to a minimum at the top of a hump that has a rise of  $\Delta z = 0.2 \text{ m}$ . Calculate the channel width at the hump that is just sufficient to choke the flow.
- 12.7 Calculate the flow depth at the hump in problem 12.5 if the channel width there is 3 m.
- 12.8 Calculate the flow depth just upstream from the hump in problem 12.5 if the channel width at the hump is 1.3 m.

12.9



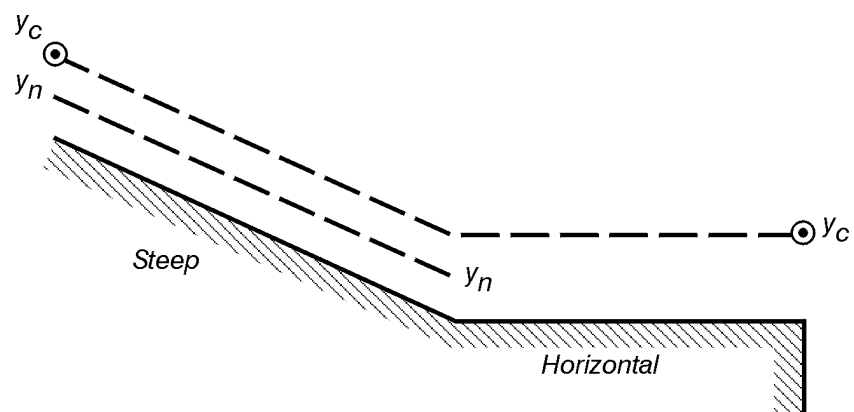
Calculate and plot the specific energy diagram,  $y$  versus  $E$ , for the channel shown above. Use the plot to determine critical depth for this flow.

12.10 Calculate the uniform flow velocity and discharge at  $10^\circ\text{C}$  for a rectangular cross section if  $B = 5 \text{ m}$ ,  $y = 2 \text{ m}$ ,  $\epsilon = 0.15 \text{ m}$  and  $S_0 = 1:3000$ .

12.11 Calculate the uniform flow depth for the channel in problem 12.10 if  $Q = 4 \text{ m}^3/\text{s}$ .

12.12 Figure 12.10 does not show the free surface profiles that result when  $y_c = y_n$ . Determine and sketch the profile behaviours for this case.

12.13



A steep channel is joined to a horizontal channel that terminates with a free overfall. Critical depth occurs at the beginning of the steep slope and at the end of the horizontal slope. Sketch and label the free surface profiles in each reach if

- a hydraulic jump occurs on the steep slope, and
- a hydraulic jump occurs on the horizontal slope.
- Explain what set of conditions would cause the jump to be swept over the overfall so that  $y < y_c$  at the brink. Show the result with a sketch.

- 12.14 Supercritical flow from a spillway enters a channel reach with a mild slope. The mild reach is joined to a very long steep reach that has negligible backwater effects. Sketch the free surface profiles that result (a) if a jump occurs in the mild reach and (b) if no jump occurs in the mild reach. Then state what conditions will cause case (b) to occur.
- 12.15 Flow exits from a spillway onto a horizontal channel with a velocity and depth of 20 m/s and 1 m, respectively. Calculate the Froude number for this flow to verify that it is supercritical. Then use one step in  $y$  and the method shown with Eq. (12.47) and Example 12.7 to calculate the distance downstream to the point where the depth is 1.1 m. Assume that  $f = 0.06$  and that the channel is wide enough to allow  $R$  to be approximated with the depth.
- 12.16 Calculate the tailwater depth that would stabilize a jump 20 m downstream in problem 12.15. Carry out the backwater calculation using one step in  $x$  and the method shown with Eqs. (12.48) - (12.49) and Example 12.8. Results from Example 4.7 will be helpful.



## CHAPTER 13

### UNSTEADY PIPE FLOW PROBLEMS

13.1 Consider the following problem:

$$x \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial t} = 1 \quad \text{for } -\infty < x < \infty \quad \text{and} \quad 0 < t < \infty$$

$$\phi(x, 0) = x \quad \text{for } -\infty < x < \infty$$

- Write the partial differential equation in characteristic form.
- Show in a sketch of the  $(x, t)$  plane the qualitative behaviour of characteristics that leave the positive  $x$  axis, the origin and the negative  $x$  axis.
- Solve the problem, obtaining both the parametric and non-parametric forms of the solution.

13.2 Consider the following problem:

$$\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial t} = t \quad \text{for } -\infty < x < \infty \quad \text{and} \quad -\infty < t < \infty$$

$$\phi(x, t) = 0 \quad \text{along } x = 1/t \quad \text{for } 0 < t < \infty$$

- Show in a sketch of the  $(x, t)$  plane the curve along which  $\phi$  is prescribed. Then use the geometry of this curve and the geometry of the characteristics to show that this prescription of initial data is sufficient to allow  $\phi$  to be calculated everywhere in the solution domain [the entire  $(x, t)$  plane].
- Calculate the problem solution.

13.3 Solve the following problem:

$$t \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial t} = e^{-t} \quad \text{for } 0 < x < \infty \quad \text{and} \quad 0 < t < \infty$$

$$\phi(x, 0) = 0 \quad \text{for } 0 \leq x < \infty$$

$$\phi(0, t) = 1/(1+t) \quad \text{for } 0 < t < \infty$$

Obtain both the parametric and non-parametric forms of the solution. Then calculate the discontinuity in  $\phi$  that occurs across the characteristic that passes through the origin in the  $(x, t)$  plane.

- 13.4 Use the method of characteristics to obtain, in both parametric and non-parametric form, the solution of the following problem:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial t} = 0 \quad \text{for } 0 < x < \infty \quad \text{and} \quad 0 < t < \infty$$

$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial t} = 0 \quad \text{for } 0 < x < \infty \quad \text{and} \quad 0 < t < \infty$$

$$u(x, 0) = \sin(x) \quad \text{for } 0 \leq x < \infty$$

$$v(x, 0) = 0 \quad \text{for } 0 \leq x < \infty$$

$$u(0, t) = \sin(t) \quad \text{for } 0 \leq t < \infty$$

- 13.5 The equations of unsteady open channel flow are

$$g \frac{\partial h}{\partial x} + U \frac{\partial U}{\partial x} + \frac{\partial U}{\partial t} = g(S_0 - S_f)$$

$$U \frac{\partial h}{\partial x} + h \frac{\partial U}{\partial x} + \frac{\partial h}{\partial t} = 0$$

in which  $g$  = gravitational constant,  $h$  = flow depth,  $U$  = flux velocity,  $x$  = distance along the channel,  $t$  = time,  $S_0$  = channel slope and  $S_f$  = friction slope given by Eq. (12.20).

- (a) Show that these two simultaneous equations with  $h$  and  $U$  as unknowns have two families of characteristics and, therefore, can be solved by using the method of characteristics. [Multiply the second equation by an unknown parameter,  $\lambda$ , add to the first equation and choose  $\lambda$  so that the derivatives of  $h$  and  $U$  are along the same curve in the  $(x, t)$  plane.]
- (b) By setting  $c = \sqrt{gh}$  and  $h = c^2/g$  in the characteristic form of the equations obtained for part (a) show that these equations become

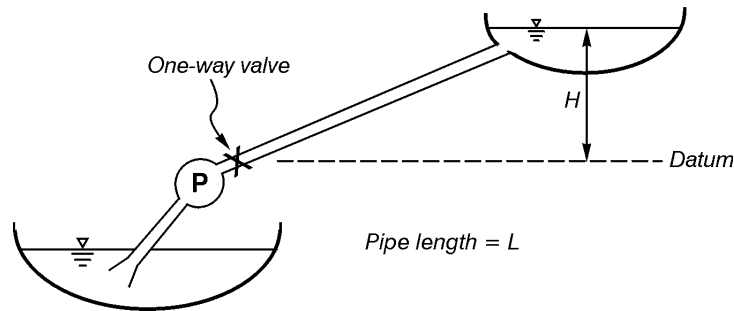
$$\frac{d(U + 2c)}{dt} = g(S_0 - S_f) \quad \text{along} \quad \frac{dx}{dt} = U + c$$

$$\frac{d(U - 2c)}{dt} = g(S_0 - S_f) \quad \text{along} \quad \frac{dx}{dt} = U - c$$

Since these equations give disturbance speeds of  $dx/dt = \pm c$  when  $U = 0$ , we see that  $c = \sqrt{gh}$  is the speed of a disturbance or wave in still water, when  $U = 0$ . Furthermore, since  $dx/dt$  is a function of the dependent variables  $U$  and  $c$ , shocks can appear in the solution of these equations.

- 13.6 Subcritical flow has  $U < c$ , and supercritical flow has  $U > c$ . Thus, subcritical open channel flow is seen from the results of problem 13.5 to have one family of characteristics sloping downstream and one family sloping upstream. On the other hand, supercritical flow has both families of characteristics sloping downstream with different slopes. Use this information to discuss what boundary and initial conditions must be prescribed to calculate unsteady open channel flow solutions in both subcritical and supercritical flows for  $0 \leq x \leq L$  and  $0 \leq t < \infty$ .

13.7

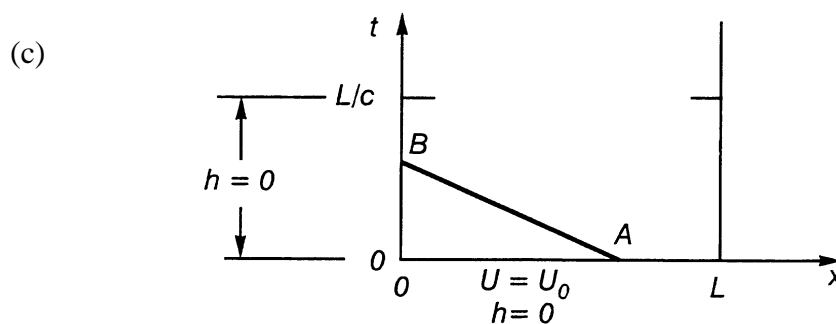


Power failure to a pump causes a valve on the upstream side of the pump to shut instantaneously and to remain shut for  $0 < t < \infty$ . Neglect all losses, and assume that maximum rises in piezometric head are very much greater than  $H$ . Calculate and plot the variations of  $h$  at the valve and of  $U$  at the entrance to the upper reservoir for  $0 \leq t \leq 8L/c$ . Notice that you can calculate  $U$  and  $h$  at the boundaries  $x = 0$  and  $x = L$  relatively easily from Eqs. (13.43 a, b) without having to calculate  $U$  and  $h$  at internal points.

13.8 The use of Eqs. (13.50) - (13.52) is relatively complicated and usually requires the use of a computer. However, some feeling for the effect of finite valve closure can be obtained by solving the following relatively simple problem:

$$\begin{aligned}
 U + gh/c &= \kappa_1 \quad \text{along } x - ct = \kappa_2 \\
 U - gh/c &= \kappa_3 \quad \text{along } x + ct = \kappa_4 \\
 h(0, t) &= 0 \quad \text{for } 0 \leq t < \infty \\
 h(x, 0) &= 0 \quad \text{for } 0 \leq x \leq L \\
 U(x, 0) &= U_0 \quad \text{for } 0 \leq x \leq L \\
 U(L, t) &= (1 - t/T) U_0 \quad \text{for } 0 \leq t \leq T \\
 &= 0 \quad \text{for } T \leq t < \infty
 \end{aligned}$$

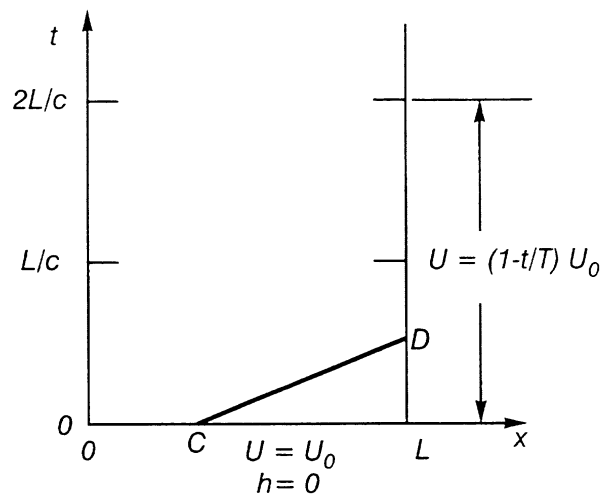
Since all boundary and initial conditions are continuous, no discontinuities in  $h$  or  $U$  will appear in the solution. Assume that  $T \geq 4L/c$ .



Use  $U - gh/c = \kappa_3$  along  $AB$  to show that

$$U(0, t) = U_0 \quad \text{for } 0 < t < L/c.$$

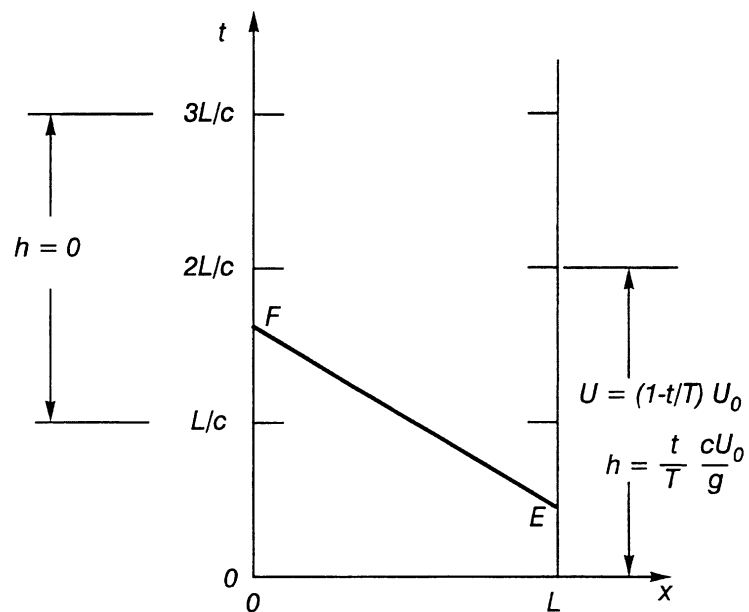
(d)



Use  $U + gh/c = \kappa_1$  along  $CD$  to show that

$$h(L, t) = \frac{t}{T} \frac{cU_0}{g} \text{ for } 0 \leq t < 2L/c.$$

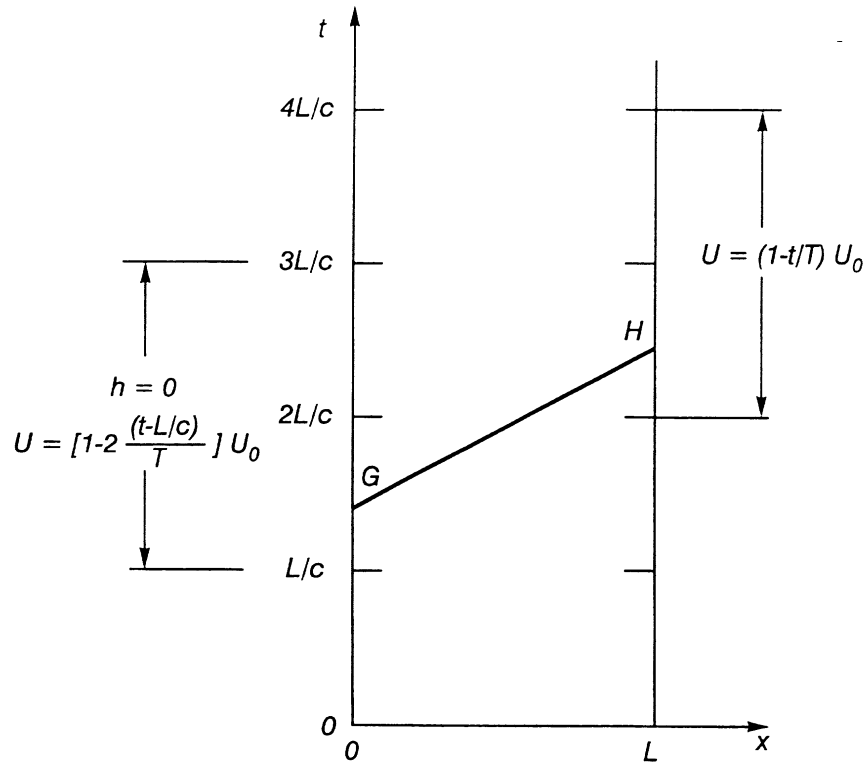
(c)



Use  $u - gh/c = \kappa_3$  and  $x + ct = \kappa_4$  along  $EF$  to show that

$$U(0, t) = \left[ 1 - 2 \frac{(t - L/c)}{T} \right] U_0 \text{ for } L/c < t < 3L/c$$

(d)



Use  $U + gh/c = \kappa_1$  and  $x - ct = \kappa_2$  along  $GH$  to show that

$$h(L, t) = \frac{(4L/c - t)}{T} \frac{cU_0}{g} \text{ for } 2L/c \leq t \leq 4L/c$$

- (e) Sketch a qualitatively correct plot of  $h(L, t)$  versus  $t$  for  $0 \leq t \leq 4L/c$ . Then comment upon the significance of the result for the maximum value of  $h$ .
- (f) Suppose that  $L = 100$  m,  $U_0 = 20$  m/s and that  $h$  has a maximum allowable value of 30 m. What is the corresponding minimum value for  $T$ ?



## CHAPTER 14

### UNSTEADY OPEN CHANNEL FLOW PROBLEMS

- 14.1 The Saint-Venant equations can be linearized to describe shallow water waves with small amplitudes in water that was initially at rest by setting  $U = U'$  and  $y = h + y'$  in which  $h =$  constant depth of the undisturbed water and  $U'$  and  $y'$  are relatively small changes in velocity and depth from the initial values of zero and  $h$ , respectively. Substitute  $U = U'$  and  $y = h + y'$  in the Saint-Venant equations with  $S_0 = S_f = 0$  and neglect second-order terms in  $U'$  and  $y'$  to show that

$$g \frac{\partial y'}{\partial x} + \frac{\partial U'}{\partial t} = 0$$

$$h \frac{\partial U'}{\partial x} + \frac{\partial y'}{\partial t} = 0$$

Then show that the characteristic form of these linearized equations is given by

$$\frac{d}{dt} \left( U + y \sqrt{\frac{g}{h}} \right) = 0 \quad \text{along} \quad \frac{dx}{dt} = \sqrt{gh}$$

$$\frac{d}{dt} \left( U - y \sqrt{\frac{g}{h}} \right) = 0 \quad \text{along} \quad \frac{dx}{dt} = -\sqrt{gh}$$

in which primes have been omitted for notational convenience.

- 14.2 A vertical wall in a semi-infinite reservoir starts to vibrate sinusoidally as a result of seismic motion. Solve the equations in problem 14.1 with the following boundary and initial conditions:

$$U(0, t) = U_m \sin(\omega t) \quad \text{for} \quad 0 \leq t < \infty$$

$$U(x, 0) = y(x, 0) = 0 \quad \text{for} \quad 0 \leq x < \infty$$

This solution, which can be used to calculate water pressures and moments on the wall and wave amplitudes in the reservoir, assumes that

- (1) Wave lengths are large compared with the undisturbed water depth,  $h$ .
- (2) Wave amplitudes are small compared with  $h$ .

Assumption (1) was made in the derivation of the Saint-Venant equations, which assumes that velocity distributions are uniform and pressure distributions are hydrostatic along vertical lines. Assumption (2) was made in linearizing the Saint-Venant equations. Obtain the wave length and amplitude from your solution and use them to put these restrictions in the form of two inequalities.

- 14.3 Flow released below a hydroelectric reservoir will gradually raise the water level to one metre above its present level. The channel slope is 1:150 and  $f = 0.08$ . Interpret the statement  $a \gg b$  to mean  $a > 10b$  to estimate the minimum time period over which the water level increase should be distributed if the kinematic wave approximation is to be used for routing the flow downstream.
- 14.4 Use the kinematic wave approximation and disregard the presence of shocks and multiple values for  $y(x, t)$  to calculate  $y$  for the following boundary condition:

$$y(0, t) = y_0(t) = 1 \quad \text{for } -\infty < t \leq 0 \text{ and } 1 \leq t < \infty$$

$$= 1 + \sin(\pi t) \text{ for } 0 \leq t \leq 1$$

Then plot  $y$  versus  $x$  on the same graph for  $t = 1/2, 2/\pi$  and  $2$  after setting

$$\frac{3}{2} \sqrt{\frac{8g}{f} S_0} = 1$$

The solution is multiple-valued for values of  $t > 2/\pi$ , and it is always easier to plot multiple-valued solutions by using the solution in its parametric form.

- 14.5 The coordinates of the kinematic wave "nose" are given by

$$t = \tau + 2 \frac{y_0(\tau)}{y_0'(\tau)}$$

$$x = 3 \frac{y_0(\tau)}{y_0'(\tau)} \sqrt{\frac{8g}{f} y_0(\tau) S_0}$$

The shock initial point occurs at the value of  $\tau$  that makes  $t$  a minimum.

- (a) Differentiate the expression for  $t$  with respect to  $\tau$ . Then discuss what types of curvature of the inflow depth hydrograph near  $\tau = 0$ , determined from the sign of  $y_0''(\tau)$ , may lead to a relative minimum for  $t$  with  $\tau > 0$  and what types of hydrograph will always cause  $t$  to achieve an absolute minimum at  $\tau = 0$ .
- (b) Calculate the shock initial point for the inflow depth hydrograph used in problem 14.4.
- 14.6 Use a graphical technique to estimate the shock location at  $t = 2$  from the solution plotted for problem 14.4.